

GEOMETRIC DESIGN OF HIGHWAYS

To demonstrate the effect of vehicle performance (specifically braking performance) and vehicle dimensions (such as driver's eye height, headlight height, and taillight height) on the design of highways. By concentrating on the specifics of the highway alignment problem, you will develop an understanding of the procedures and compromises inherent in the design of all highway-related geometric elements.

INTRODUCTION

The design of highways necessitates the determination of specific design elements, which include:

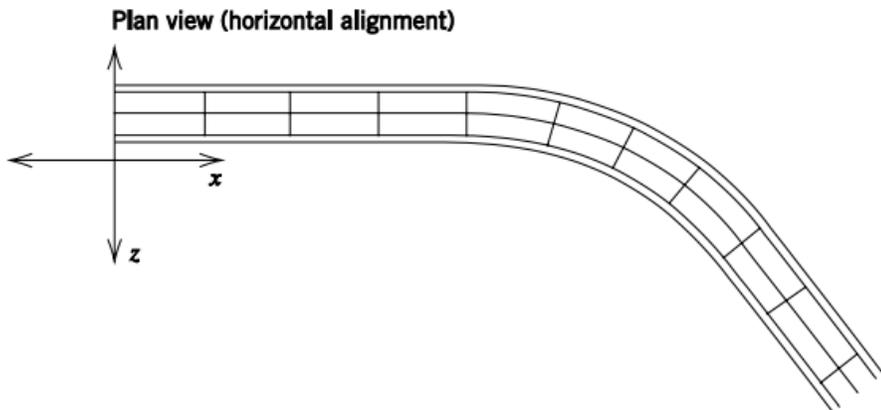
- The number of lanes, lane width, median type (if any) and width length of acceleration and deceleration
- Lanes for on- and off-ramps
- Curve radii required for vehicle turning
- The alignment required to provide adequate stopping and passing sight distances

PRINCIPLES OF HIGHWAY ALIGNMENT

- The alignment of a highway is a three-dimensional problem measured in **x, y, and z coordinates**.
- Three-dimensional design computations are **complex**, and the actual implementation and construction of a design based on three-dimensional coordinates has historically been **prohibitively difficult**.
- The three-dimensional highway alignment problem is reduced to **two two-dimensional** alignment problems

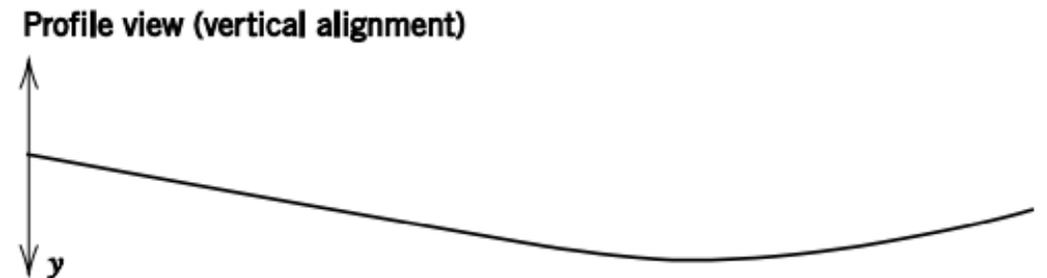
1. Horizontal Alignment

The horizontal alignment of a highway is referred to as the plan view, which is roughly equivalent to the perspective of an aerial photo of the highway.



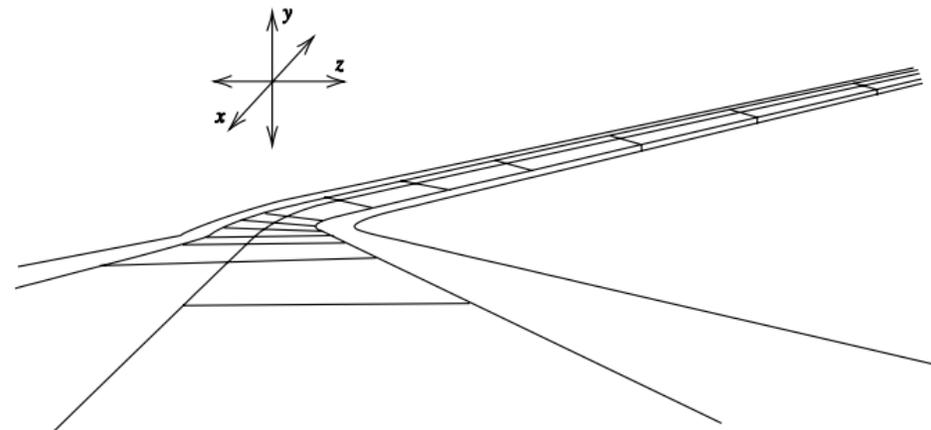
2. Vertical Alignment

The vertical alignment is represented in a profile view, which gives the elevation of all points measured along the length of the highway (again, with length measured along a constant elevation reference).



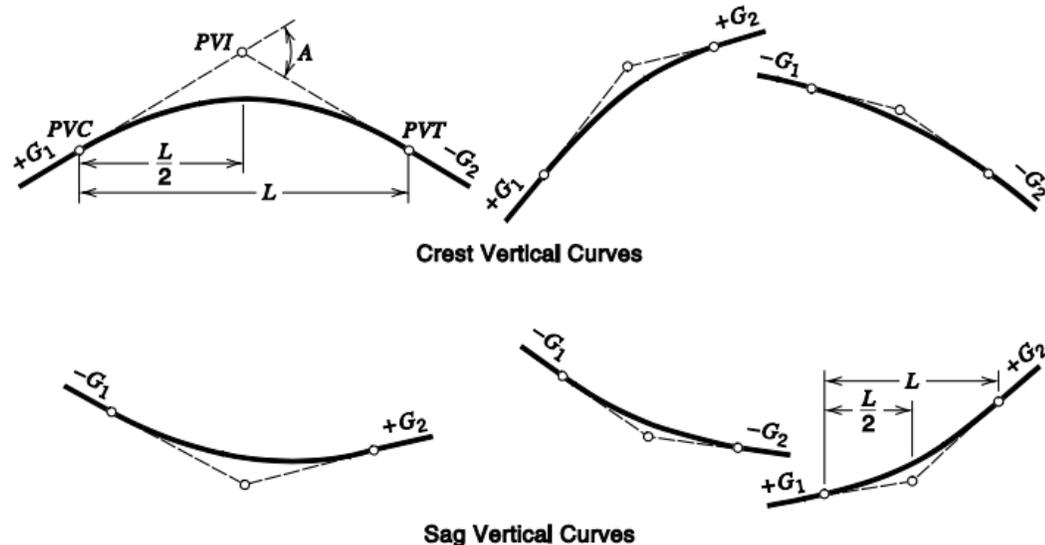
PRINCIPLES OF HIGHWAY ALIGNMENT

- Instead of using x and z coordinates, highway positioning and length are defined as the distance along the highway (usually measured along the centerline of the highway, on a horizontal, constant- elevation plane) from a specified point.
- The distance is measured in terms of stations, with each station consisting of **100 ft** of highway alignment distance.
 - Ex: The notation for stationing distance is such that a point on a highway 4250 ft from a specified point of origin is said to be at station 42 + 50 ft, that is, 42 stations and 50 ft, with the point of origin being at station 0 + 00.
- This stationing concept, combined with the highway's alignment direction given in the **plan view** (horizontal alignment) and the elevation corresponding to stations given in the **profile view** (vertical alignment)



VERTICAL ALIGNMENT

- The elevation of roadway points is usually determined by the need to provide an acceptable level of **driver safety, driver comfort, and proper drainage**
- A primary concern in vertical alignment is establishing the transition of roadway elevations between two grades. This transition is achieved by means of a vertical curve.
- Vertical curves can be broadly classified into **crest vertical curves** and **sag vertical curves**
- Sag curves are used where the change in grade is positive, such as valleys, while crest curves are used when the change in grade is negative, such as hills.
- Each roadway point is **uniquely defined by stationing** (which is measured along a horizontal plane) and elevation.



G_1 = initial roadway grade in percent or ft/ft (this grade is also referred to as the initial tangent grade, viewing Fig. 3.3 from left to right),

G_2 = final roadway (tangent) grade in percent or ft/ft,

A = absolute value of the difference in grades (initial minus final, usually expressed in percent),

L = length of the curve in stations or ft measured in a constant-elevation horizontal plane.

PVC = point of the vertical curve (the initial point of the curve),

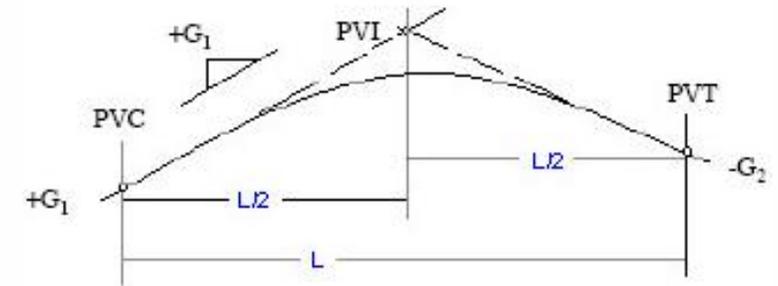
PVI = point of vertical intersection (intersection of initial and final grades), and

PVT = point of vertical tangent, which is the final point of the vertical curve (the point where the curve returns to the final grade or, equivalently, the final tangent).

VERTICAL CURVE FUNDAMENTALS

- The general form of the parabolic equation is defined below, where y is the elevation for the parabola

$$y = ax^2 + bx + c$$



- At $x = 0$, which refers to the position along the curve that corresponds to the PVC, the elevation equals the elevation of the PVC. Thus, the value of c equals e_{PVC} .

$$y = a(0)^2 + b(0) + c \implies c = e_{PVC}$$

- The first derivative of the equation gives the slope, and the slope of the curve at $x = 0$ equals the incoming slope at the PVC, or G_1 .

$$\frac{dy}{dx} = 2ax + b \quad \therefore \frac{dy}{dx} = 2a(0) + b \implies b = G_1$$

- The second derivative, which equals the rate of slope change, and the average rate of change of slope can also be written as

$$\frac{d^2y}{dx^2} = 2a \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{G_2 - G_1}{L} \implies a = \frac{G_2 - G_1}{2L}$$

VERTICAL CURVE FUNDAMENTALS

- **Offsets**, which are vertical distances from the initial tangent to the curve, play a significant role in vertical curve design. The formula for determining offset is listed below.

$$Y = \frac{A}{200L}x^2 \implies Y_m = \frac{AL}{800} \quad \text{and} \quad Y_f = \frac{AL}{200}$$

A = absolute value of the difference in grades $|G_1 - G_2|$ expressed in percent

- **K-value** can be used to compute the high and low point locations of crest and sag vertical curves, respectively by knowing the horizontal distance, in ft, required to affect a 1% change in the slope of the vertical curve.

$$x_{hl} = K|G_1| = \frac{L}{A}|G_1|$$

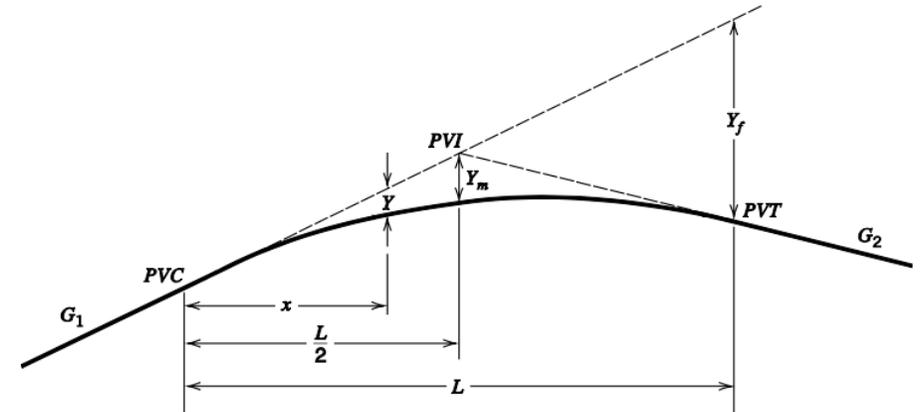


Figure 3.4 Offsets for equal-tangent vertical curves.

- | | |
|---|---|
| G_1 = initial roadway grade in percent or ft/ft (this grade is also referred to as the initial tangent grade, viewing Fig. 3.4 from left to right), | x = distance from the PVC in ft, |
| G_2 = final roadway (tangent) grade in percent or ft/ft, | L = length of the curve in stations or ft measured in a constant-elevation horizontal plane, |
| Y = offset at any distance x from the PVC in ft, | PVC = point of the vertical curve (the initial point of the curve), |
| Y_m = midcurve offset in ft, | PVI = point of vertical intersection (intersection of initial and final grades), and |
| Y_f = offset at the end of the vertical curve in ft, | PVT = point of vertical tangent, which is the final point of the vertical curve (the point where the curve returns to the final grade or, equivalently, the final tangent). |

EXAMPLE 3.1

A **600-ft** equal-tangent sag vertical curve has the **PVC at station 170 + 00** and **elevation 1000 ft**. The initial grade is **3.5%** and the final grade is **+0.5%**. Determine the stationing and elevation of the PVI, the PVT, and the lowest point on the curve.

SOLUTION

1) Since the curve is equal tangent, the PVI will be 300 ft or three stations (measured in a horizontal plane) from the PVC, and the PVT will be 600 ft or six stations from the PVC.

- **the stationing of the PVI is 173 + 00**
- **the stationing of the PVT is 176 + 00.**

2) the elevations of the PVI and PVT, it is known that a 3.5% grade can be equivalently written as 3.5 ft/station (a 3.5 ft drop per 100 ft of horizontal distance). Since the PVI is three stations from the PVC, which is known to be at elevation 1000 ft

$$\text{The elevation of the PVI} = e_{PVC} - (G)(L/2) = 1000 - (0.035)(300) = 989.5 \text{ ft}$$

$$\text{The elevation of the PVT} = 989.5 + 0.5(\text{ft/station})3(\text{stations}) = 989.5 + (0.005)(300) = 991 \text{ ft}$$

EXAMPLE 3.1

A **600-ft** equal-tangent sag vertical curve has the **PVC at station 170 + 00** and **elevation 1000 ft**. The initial grade is **3.5%** and the final grade is **+0.5%**. Determine the stationing and elevation of the PVI, the PVT, and the lowest point on the curve.

SOLUTION

3) When initial and final grades are not opposite in sign, the low (or high) point on the curve will not be where the first derivative is zero because the slope along the curve will never be zero

$$y = ax^2 + bx + c \implies \frac{dy}{dx} = 0 = 2ax + b \quad \text{and} \quad b = G_1$$

$$a = \frac{G_2 - G_1}{2L} = \frac{0.5 - (-3.5)}{2(6)} = 0.33333$$

$$\frac{dy}{dx} = 0 = 2ax + b = 2(0.333)x + (-3.5) \implies x = 5.25 \text{ stations}$$

- the stationing of the low point at 175 + 25 (5 + 25 stations from the PVC).
the elevation of the lowest point on the vertical curve.

$$y = ax^2 + bx + c \implies y = 0.33333(5.25)^2 + (3.5)(5.25) + 1000 = 990.81 \text{ ft}$$

EXAMPLE 3.4

A vertical curve crosses a 4-ft diameter pipe at right angles. The pipe is located at **station 110 + 85** and its centerline is at elevation **1091.60 ft**. The PVI of the vertical curve is at station **110 + 00** and elevation **1098.4 ft**. The vertical curve is **equal tangent, 600 ft long**, and connects an initial grade of **+1.20%** and a final grade of **1.08%**. Using offsets, determine the depth, below the surface of the curve, of the top of the pipe and determine the station of the highest point on the curve.

SOLUTION

1) Since the curve is equal tangent, the PVC will be $(660/2)$ ft or three stations (measured in a horizontal plane) from the PVI

- **The PVC is at station = $(110 + 00) - (3 + 00) = (107 + 00)$ ft**
- **The pipeline is at station = $(110 + 85) - (107 + 00) = (003 + 85)$ ft = 385 ft**

2) the elevations of the PVC and the pipeline:

$$\text{The elevation of the PVC} = 1098.4 - [1.2(\text{ft/station})3(\text{stations})] = 1098.4 - [(0.012)(300)] = 1094.8 \text{ ft}$$

$$\text{The elevation of the pipe} = 1098.4 + [1.2(\text{ft/station})3.85(\text{stations})] = 1098.4 - [(0.0385)(300)] = 1099.42 \text{ ft}$$

EXAMPLE 3.4

A vertical curve crosses a 4-ft diameter pipe at right angles. The pipe is located at **station 110 + 85** and its centerline is at elevation **1091.60 ft**. The PVI of the vertical curve is at station **110 + 00** and elevation **1098.4 ft**. The vertical curve is **equal tangent, 600 ft long**, and connects an initial grade of **+1.20%** and a final grade of **1.08%**. Using offsets, determine the depth, below the surface of the curve, of the top of the pipe and determine the station of the highest point on the curve.

SOLUTION

3) determine the offset above the pipe at $x = 385$ ft (the distance of the pipe from the PVC)

$$Y = \frac{A}{200L} x^2 = \frac{|1.2 - (-1.08)|}{200(600)} (385)^2 = 2.82 \text{ ft}$$

- The elevation of the curve above the pipe = $1099.42 - 2.82 = 1096.6$ ft
- The elevation of the top of the pipe = $1091.60 + 2 = 1093.60$ ft
- the pipe below the surface of the curve = $1096.6 - 1093.6 = 3$ ft

EXAMPLE 3.4

A vertical curve crosses a 4-ft diameter pipe at right angles. The pipe is located at **station 110 + 85** and its centerline is at elevation **1091.60 ft**. The PVI of the vertical curve is at station **110 + 00** and elevation **1098.4 ft**. The vertical curve is **equal tangent, 600 ft long**, and connects an initial grade of **+1.20%** and a final grade of **1.08%**. Using offsets, determine the depth, below the surface of the curve, of the top of the pipe and determine the station of the highest point on the curve.

SOLUTION

4) To determine the location of the highest point on the curve, we find K the multiples it by the absolute of G_1

$$x_{hl} = K|G_1| = \frac{L}{A}|G_1| = \frac{600}{|1.2 - (-1.08)|}|1.2| = 315.79 \text{ ft}$$

- The station of the highest point = $107 + 00 + 3 + 15.79 = \mathbf{110 + 15.79 \text{ ft}}$
- Note that this example could also be solved by following the procedure used in Example 3.1.

STOPPING SIGHT DISTANCE

- the primary challenges facing highway designers is to minimize construction costs (usually by making the vertical curve as short as possible) while still providing an adequate level of safety.
- An appropriate level of safety is the level of safety that gives drivers sufficient sight distance to allow them to safely stop their vehicles to avoid collisions with objects obstructing their forward motion.

$$SSD = \frac{V_1^2}{2g \left(\left(\frac{a}{g} \right) \pm G \right)} + V_1 \times t_r \quad (3.12)$$

where

- SSD = stopping sight distance in ft,
- V_1 = initial vehicle speed in ft/s,
- g = gravitational constant, 32.2 ft/s²,
- a = deceleration rate in ft/s²,
- G = roadway grade (+ for uphill and – for downhill) in percent/100, and
- t_r = perception/reaction time in s.

Table 3.1 Stopping Sight Distance

Design speed (mi/h)	Brake reaction distance (ft)	Braking distance on level (ft)	Stopping sight distance	
			Calculated (ft)	Design (ft)
15	55.1	21.6	76.7	80
20	73.5	38.4	111.9	115
25	91.9	60.0	151.9	155
30	110.3	86.4	196.7	200
35	128.6	117.6	246.2	250
40	147.0	153.6	300.6	305
45	165.4	194.4	359.8	360
50	183.8	240.0	423.8	425
55	202.1	290.3	492.4	495
60	220.5	345.5	566.0	570
65	238.9	405.5	644.4	645
70	257.3	470.3	727.6	730
75	275.6	539.9	815.5	820
80	294.0	614.3	908.3	910

STOPPING SIGHT DISTANCE AND CREST VERTICAL CURVE DESIGN

- The length of curve (L) is the critical element in providing sufficient SSD on a vertical curve.
- Longer curve lengths provide more SSD, all else being equal, but are more costly to construct.
- Shorter curve lengths are less expensive to construct but may not provide adequate SSD due to more rapid changes in slope.
- Expression for minimum curve length given a required SSD:

For $S < L$

$$L_m = \frac{AS^2}{200(\sqrt{H_1} + \sqrt{H_2})^2}$$

For $SSD < L$

$$L_m = \frac{A \times SSD^2}{2158} \quad (3.15)$$

For $S > L$

$$L_m = 2S - \frac{200(\sqrt{H_1} + \sqrt{H_2})^2}{A}$$

For $SSD > L$

$$L_m = 2 \times SSD - \frac{2158}{A} \quad (3.16)$$

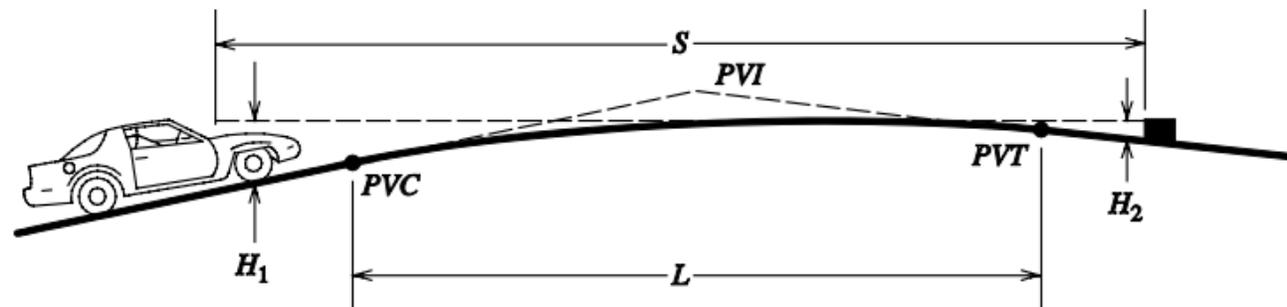


Figure 3.6 Stopping sight distance considerations for crest vertical curves.

EXAMPLE 3.5

A highway is being designed to AASHTO guidelines with a **70-mi/h** design speed, and at one section, an **equal-tangent vertical curve** must be designed to connect grades of **+1.0%** and **2.0%**. Determine the minimum length of curve necessary to meet SSD requirements.

SOLUTION

If we ignore the effect of grades ($G_s = 0$), the SSD can be read directly from Table 3.1. In this case, the SSD corresponding to a speed of 70 mi/h is 730 ft. If we assume that $L > SSD$:

$$L_m = \frac{A \times SSD^2}{2158} = \frac{3 \times 730^2}{2158} = \underline{\underline{740.82 \text{ ft}}}$$

Since 740.82 > 730 ft, the assumption that L > SSD was correct.

STOPPING SIGHT DISTANCE AND CREST VERTICAL CURVE DESIGN

- If **G does not equal 0**, we cannot use the SSD values in Table and instead must apply SSD equation with the appropriate G value.
- Some design agencies ignore the effect of grades completely, while others assume G is equal to **zero for grades less than 3%** and use simple adjustments to the SSD, depending on the initial and final grades, for grades of 3% or more.
- For the remainder of this chapter, we will mostly ignore the effect of grades
- The relationship between A and L_m is linear, always when $L > SSD$

$$L_m = KA = \frac{(A)(SSD)^2}{2158} \implies K = \frac{(SSD)^2}{2158}$$

Table 3.2 Design Controls for Crest Vertical Curves Based on Stopping Sight Distance

Design speed (mi/h)	Stopping sight distance (ft)	Rate of vertical curvature, K^*	
		Calculated	Design
15	80	3.0	3
20	115	6.1	7
25	155	11.1	12
30	200	18.5	19
35	250	29.0	29
40	305	43.1	44
45	360	60.1	61
50	425	83.7	84
55	495	113.5	114
60	570	150.6	151
65	645	192.8	193
70	730	246.9	247
75	820	311.6	312
80	910	383.7	384

A common alternative to these limits is to set the minimum curve length limit at three times the design speed

EXAMPLE 3.6

A highway is being designed to AASHTO guidelines with a **70-mi/h** design speed, and at one section, an **equal-tangent vertical curve** must be designed to connect grades of **+1.0%** and **-2.0%**. Determine the minimum length of curve necessary to meet SSD requirements using the K-values in Table

SOLUTION

A = 3, 70-mi/h design speed, then K = 247 (from Table 3.2)

$$L_m = KA = (247)(3) = 741.00 \text{ ft}$$

- almost identical to the 740.82 ft obtained in Example 3.5. This difference is due to rounding.

Table 3.2 Design Controls for Crest Vertical Curves Based on Stopping Sight Distance

Design speed (mi/h)	Stopping sight distance (ft)	Rate of vertical curvature, K^*	
		Calculated	Design
15	80	3.0	3
20	115	6.1	7
25	155	11.1	12
30	200	18.5	19
35	250	29.0	29
40	305	43.1	44
45	360	60.1	61
50	425	83.7	84
55	495	113.5	114
60	570	150.6	151
65	645	192.8	193
70	730	246.9	247
75	820	311.6	312
80	910	383.7	384

EXAMPLE 3.7

If the grades in Example 3.5 intersect at station 100 + 00, determine the stationing of the PVC, PVT, and curve high point for the minimum curve length based on SSD requirements.

SOLUTION

- Using the curve length from Example 3.6, $L = 741$ ft.
- The curve is equal tangent, so one-half of the curve will occur before the PVI and one-half after.

$$PVC = (100 + 00) - L/2 = (100 + 00) - (3 + 70.5) = 96 + 29.5 ft$$

$$PVT = (100 + 00) + (L/2) = (100 + 00) + (3 + 70.5) = 103 + 70.5 ft$$

- The stationing of the high point

$$x_{hl} = K|G_1| = (247)(1) = 247 \text{ ft or. } 98 + 76.5 \text{ ft}$$

EXERCISE

A +2% grade intersects a -4% grade at station 200+00 at an elevation 30 m. Determine the length of the vertical connects the two grades based on SSD, if the design speed = 75 kph then determine station of PVC & PVT

SOLUTION

STOPPING SIGHT DISTANCE AND SAG VERTICAL CURVE DESIGN

- Sag vertical curve design differs from crest vertical curve design in the sense that sight distance is governed by **nighttime conditions**
- The critical concern for sag vertical curve design is the length of roadway illuminated by the vehicle headlights
- To determine the minimum length of curve for a required sight distance, the properties of a parabola for an equal-tangent curve can be used to show that

For $S < L$

$$L_m = \frac{AS^2}{200(H + S \tan \beta)}$$

For $SSD < L$

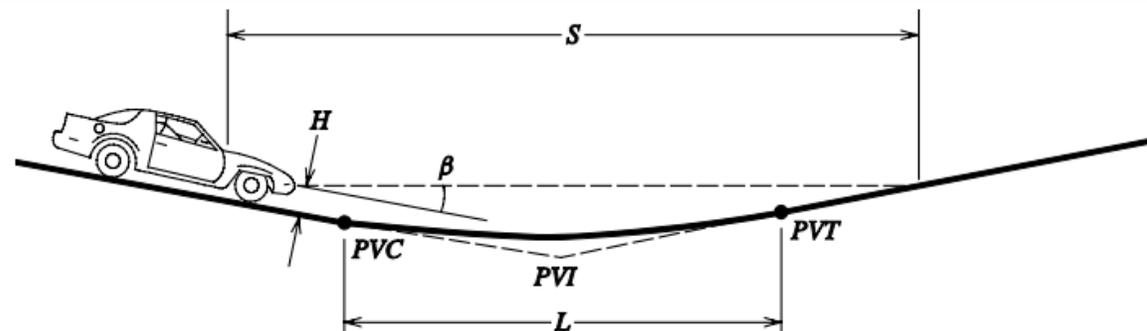
$$L_m = \frac{A \times SSD^2}{400 + 3.5 \times SSD}$$

For $S > L$

$$L_m = 2S - \frac{200(H + S \tan \beta)}{A}$$

For $SSD > L$

$$L_m = 2 \times SSD - \frac{400 + 3.5 \times SSD}{A}$$



STOPPING SIGHT DISTANCE AND SAG VERTICAL CURVE DESIGN

- As was the case for crest vertical curves, K-values can be computed by assuming $L > SSD$, which gives us the linear relationship between L_m and A . Thus for sag vertical curves (with $L_m = KA$),
- The K-values corresponding to design-speed-based SSDs are presented in Table 3.3.
- As was the case for crest vertical curves, some caution should be exercised in using this table because the assumption that $G = 0$ (for determining SSD) is used. Also, assume that $L > SSD$ is a safe, conservative assumption

$$L_m = KA = \frac{SSD^2}{400 + 3.5SSD}$$

Table 3.3 Design Controls for Sag Vertical Curves Based on Stopping Sight Distance

Design speed (mi/h)	Stopping sight distance (ft)	Rate of vertical curvature, K^*	
		Calculated	Design
15	80	9.4	10
20	115	16.5	17
25	155	25.5	26
30	200	36.4	37
35	250	49.0	49
40	305	63.4	64
45	360	78.1	79
50	425	95.7	96
55	495	114.9	115
60	570	135.7	136
65	645	156.5	157
70	730	180.3	181
75	820	205.6	206
80	910	231.0	231

EXAMPLE 3.8

An equal tangent sag vertical curve has an initial grade of -2.5% . It is known that the final grade is positive and that the low point is at elevation **270 ft** and station **141+00**. The PVT of the curve is at elevation **274 ft** and the design speed of the curve is **35 mi/h**. **Determine the station and elevation of the PVC and PVI.**

SOLUTION

- From Table 3.3 it can be seen the $K = 49$ for a design speed of 35 mi/h. With this, equation 3.11 is used to find the distance of the low point from the PVC:

$$x_{hl} = K \times |G_1| = 49(2.5) = 122.5 \text{ ft}$$

- Knowing the elevation of the low point (270 ft) and the distance of the low point from the PVC (122.5 ft), the general equation can be applied to determine the elevation of the PVC (c):

$$y = ax^2 + bx + c \implies b = G_1, \quad a = \frac{(G_2 - G_1)}{2L}, \quad c = e_{PVC}$$

- Because the final grade is known to be positive (and with G_2 being negative)

$$A = |G_1 - G_2| = G_2 - G_1$$

EXAMPLE 3.8

An equal tangent sag vertical curve has an initial grade of -2.5% . It is known that the final grade is positive and that the low point is at elevation **270 ft** and station **141+00**. The PVT of the curve is at elevation **274 ft** and the design speed of the curve is **35 mi/h**. **Determine the station and elevation of the PVC and PVI.**

SOLUTION

- Using $L = KA$

$$a = \frac{G_2 - G_1}{2L} = \frac{A/100}{2KA} = \frac{0.01}{2K} = \frac{0.01}{2(49)} = 0.000102$$

- At the low point, $y = 270$ ft so solving for c in Eq. 3.1 with $x = 122.5$, $a = 0.000102$ and $b = 0.025$ gives

$$\begin{aligned} 270 &= 0.000102(122.5)^2 + (-0.025)(122.5) + c \\ c &= \underline{\underline{271.53}} = \text{elevation of the PVC} \end{aligned}$$

- Knowing that the station of the low point is 141+00 and the distance from the PVC to the low point is 122.5 ft

$$\text{station of PVC} = 141 + 00 \text{ minus } 1 + 22.5 = \underline{\underline{139 + 77.5}}$$

EXAMPLE 3.8

An equal tangent sag vertical curve has an initial grade of **-2.5%**. It is known that the final grade is positive and that the low point is at elevation **270 ft** and station **141+00**. The PVT of the curve is at elevation **274 ft** and the design speed of the curve is **35 mi/h**. **Determine the station and elevation of the PVC and PVI.**

SOLUTION

- The length of the curve is determined by applying the general equation. Because it is known that the elevation of the PVT is 274 ft, using $y = 274$ means that $x = L$ (with $c = 271.53$)

$$y = aL^2 + bL + c$$

$$274 = 0.000102L^2 + (-0.025)L + 271.53$$

$$L = \underline{\underline{320.624 \text{ ft}}}$$

The station of the *PVI* is

$$\begin{aligned} \text{station of } PVI &= \text{station of } PVC + L/2 \\ &= 139+77.5 \text{ plus } (320.624/2) = \underline{\underline{141+37.812}} \end{aligned}$$

Finally, the elevation of the *PVI* is determined as

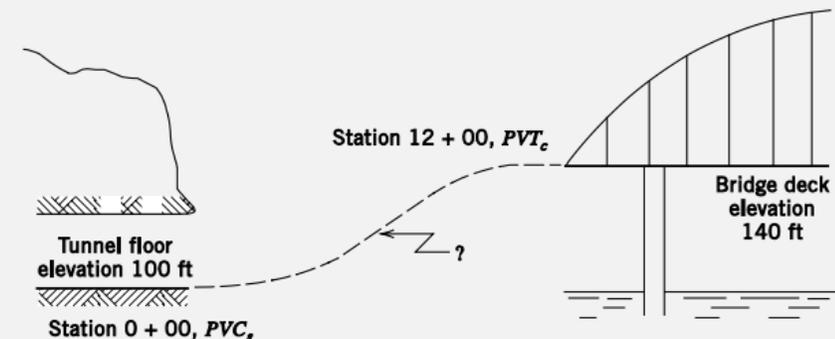
$$\begin{aligned} \text{elevation of } PVI &= \text{elevation of } PVC + G_1(L/2) \\ &= 271.53 - 0.025(320.624/2) = \underline{\underline{267.52}} \end{aligned}$$

EXAMPLE 3.9

An existing tunnel needs to be connected to a newly constructed bridge with sag and crest vertical curves. The profile view of the tunnel and bridge is shown in Fig. Develop a vertical alignment to connect the tunnel and bridge by determining the highest possible common design speed for the sag and crest (equal-tangent) vertical curves needed. **Compute the stationing and elevations of PVC, PVI, and PVT curve points.**

SOLUTION

- From the given information, it is known that $G_{1s} = 0\%$ (the initial slope of the sag vertical curve) and $G_{2c} = 0\%$ (the final slope of the crest vertical curve).
- To obtain the highest possible design speed, we want to use all of the horizontal distance available.
- This means we want to connect the curve so that the PVT of the sag curve (PVT_s) will be the PVC of the crest curve (PVC_c). If this is the case, $G_{2s} = G_{1c}$ and since $G_{1s} = G_{2c} = 0$, $A_s = A_c = A$



EXAMPLE 3.9

An existing tunnel needs to be connected to a newly constructed bridge with sag and crest vertical curves. The profile view of the tunnel and bridge is shown in Fig. Develop a vertical alignment to connect the tunnel and bridge by determining the highest possible common design speed for the sag and crest (equal-tangent) vertical curves needed. **Compute the stationing and elevations of PVC, PVI, and PVT curve points.**

SOLUTION

Solving for A gives $A = 6.667\%$. The problem now becomes one of finding K -values that allow $L_s + L_c = 1200$. Since $L = KA$ (Eq. 3.17), we can write

$$K_s A + K_c A = 1200$$

Substituting $A = 6.667$,

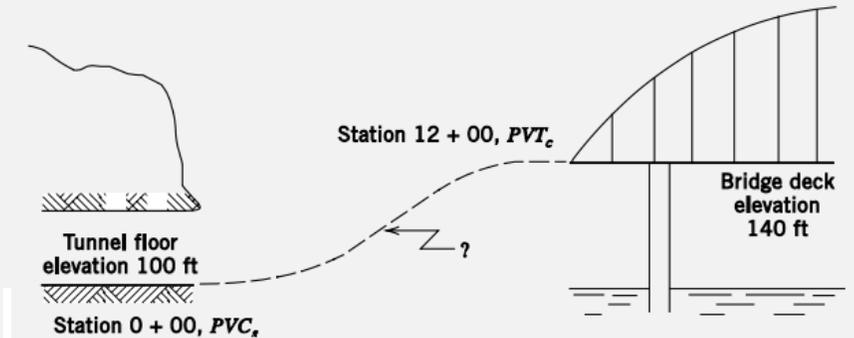
$$K_s + K_c = 180$$

To find the highest possible design speed, Tables 3.2 and 3.3 are used to arrive at K -values to solve $K_s + K_c = 180$. From Tables 3.2 and 3.3 it is apparent that the highest possible design speed is 50 mi/h, at which speed $K_c = 84$ and $K_s = 96$ (the summation of K 's is 180).

To arrive at the stationing of curve points, we first determine curve lengths as

$$L_s = K_s A = 96(6.667) = 640.0 \text{ ft}$$

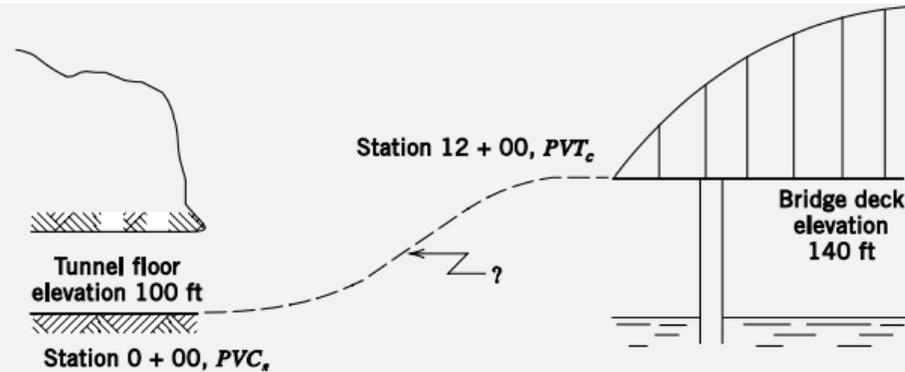
$$L_c = K_c A = 84(6.667) = 560.0 \text{ ft}$$



EXAMPLE 3.9

An existing tunnel needs to be connected to a newly constructed bridge with sag and crest vertical curves. The profile view of the tunnel and bridge is shown in Fig. Develop a vertical alignment to connect the tunnel and bridge by determining the highest possible common design speed for the sag and crest (equal-tangent) vertical curves needed. **Compute the stationing and elevations of PVC, PVI, and PVT curve points.**

SOLUTION



Since the station of the PVC_s is $0 + 00$ (given), it is clear that the $PVI_s = 3 + 20.0$, $PVT_s = PVC_c = 6 + 40.0$, $PVI_c = 9 + 20.0$, and $PVT_c = 12 + 00.0$. For elevations, $PVC_s = PVI_s = 100$ ft and $PVI_c = PVT_c = 140$ ft. Finally, the elevation of PVT_s and PVC_c can be computed as

$$100 + \frac{AL_s}{200} = 100 + \frac{6.667(640.0)}{200} = 121.33 \text{ ft}$$

PASSING SIGHT DISTANCE AND CREST VERTICAL CURVE DESIGN

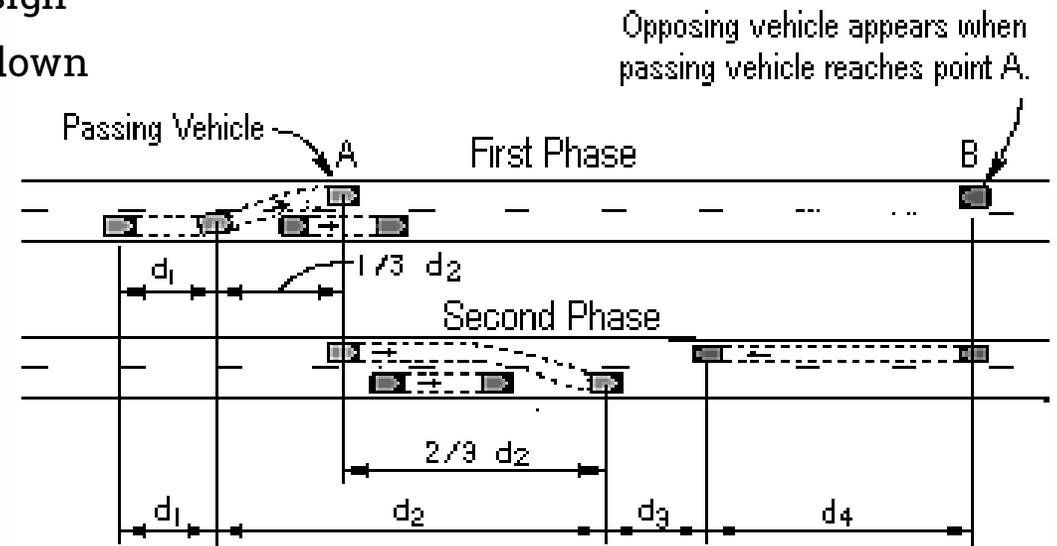
- it may be desirable to provide adequate passing sight distance, which can be an important issue in **two-lane highway** design (one lane in each direction).
- Passing sight distance is a factor only in crest vertical curve design
- for sag curves, the sight distance is unobstructed looking up or down the grade, and at night, the headlights of oncoming or opposing vehicles will be seen.

For $PSD < L$

$$L_m = \frac{A \times PSD^2}{2800}$$

For $PSD > L$

$$L_m = 2 \times PSD - \frac{2800}{A}$$



- As was the case for stopping sight distance, it is typically assumed that the length of curve is greater than the required sight distance (in this case $L > PSD$)

$$K = \frac{PSD^2}{2800}$$

PASSING SIGHT DISTANCE AND CREST VERTICAL CURVE DESIGN

The passing sight distance (PSD) used for design is assumed to consist of four distances:

- (d_1) the initial maneuver distance (which includes the driver's perception/reaction time and the time it takes to bring the vehicle from its trailing speed to the point of encroachment on the left lane)
- (d_2) the distance that the passing vehicle traverses while occupying the left lane,
- (d_3) the clearance length between the passing and opposing vehicles at the end of the passing maneuver
- (d_4) the distance traversed by the opposing vehicle during two-thirds of the time the passing vehicle occupies the left lane.

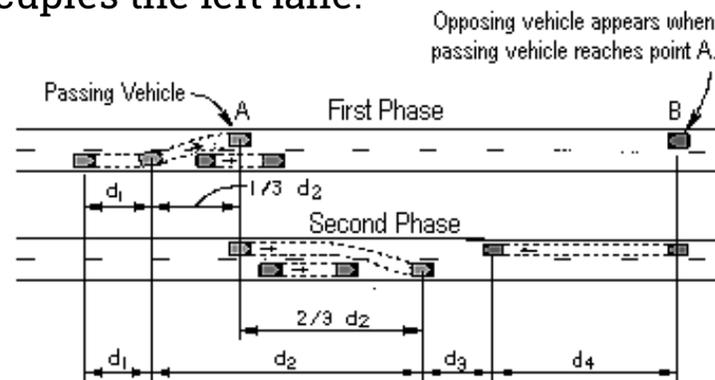


Table 3.4 Design Controls for Crest Vertical Curves Based on Passing Sight Distance.

Design speed (mi/h)	Passing sight distance (ft)	Rate of vertical curvature, K^*
20	400	57
25	450	72
30	500	89
35	550	108
40	600	129
45	700	175
50	800	229
55	900	289
60	1000	357
65	1100	432
70	1200	514
75	1300	604
80	1400	700

EXAMPLE 3.12

An equal-tangent crest vertical curve is 1000 ft long and connects a **+2.5%** and **1.5%** grade. If the design speed of the roadway is **55 mi/h**, **does this curve have adequate passing sight distance?**

SOLUTION

- To determine the length of curve required to provide adequate passing sight distance at a design speed of 55 mi/h, we use $L = KA$ with $K = 289$ (as read from Table 3.4). This give

$$L = 289(4.0) = 1156 \text{ ft}$$

- Since the curve is only 1000 ft long, it is not long enough to provide adequate passing sight distance.
- The K-value for the existing design can be compared with that required for a PSD-based design. The K-value for the existing design is

$$K = \frac{1000}{4} = 250$$

Since the K-value of 250 for the existing curve design is less than 289, this curve does not provide adequate PSD for a 55-mi/h design speed.

UNDERPASS SIGHT DISTANCE AND SAG VERTICAL CURVE DESIGN

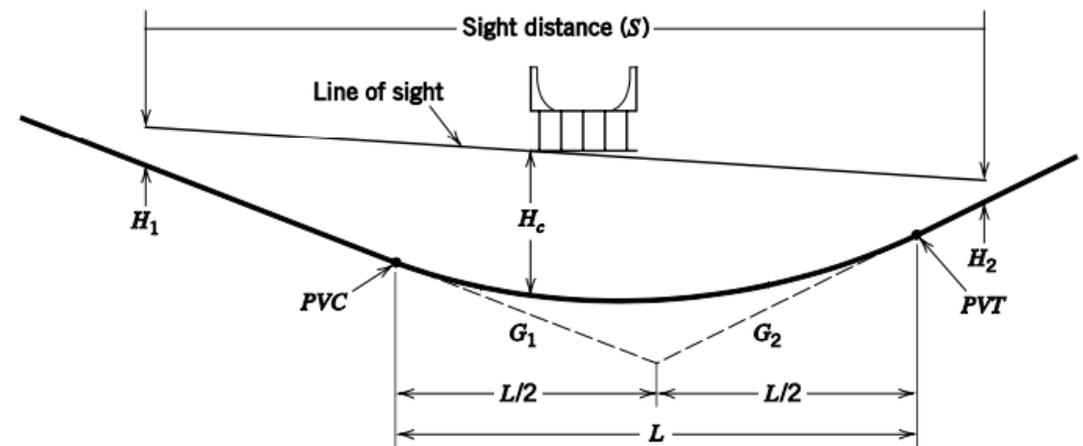
- Design for sag curves is based on nighttime conditions because during daytime conditions a driver can see the entire sag curve.
- in the case of a sag curve being built under an overhead structure (such as roadway or railroad crossing), a driver's line of sight may be restricted so that the entire curve length is not visible.
- it is essential that the curve be long enough to provide a suitably gradual rate of curvature such that the overhead structure does not block the line of sight and allows the required stopping sight distance for the specified design speed to be maintained
- AASHTO [2011] recommends a minimum structure clearance height of 14.5 ft and a desirable clearance height of 16.5 ft. also recommends that clearance heights be no less than 1 ft greater than the maximum allowable vehicle height.
- in building a new overpass structure over an existing sag curve alignment, the clearance height must be determined for both required **stopping sight distance** and **maximum allowable vehicle height** for that roadway. and the greater of the two values should be used.

For $SSD < L$

$$L = \frac{A \times SSD^2}{800(H_c - 5)}$$

For $SSD > L$

$$L = 2 \times SSD - \frac{800(H_c - 5)}{A}$$



EXAMPLE 3.13

An equal-tangent sag curve has an initial grade of **4.0%**, a final grade of **+3.0%**, and a length of **1270 ft**. An overpass is being placed directly over the PVI of this curve. At what height above the roadway should the bottom of this sign be placed?

SOLUTION

- The design speed for the curve can be determined from the K-value by applying Eq. 3.10 as follows
- from Table 3.3, this K-value corresponds approximately to a design speed of **70 mi/h (K = 181)**. For a 70-mi/h design speed, the required stopping sight distance is **730 ft**.
- Since the curve length is greater than the required SSD ($1270 > 730$), Eq. 3.29 applies:
- Solve for the clearance height, H_c , and substituting $A = 7\%$, $SSD = 730$ ft, and $L = 1270$ ft gives
- Although only 8.67 ft is needed for SSD requirements. Though, the bottom of the overpass should be placed at least 14.5 ft above the roadway surface (at the PVI), or desirably at a height of 16.5 ft according to AASHTO [2011].

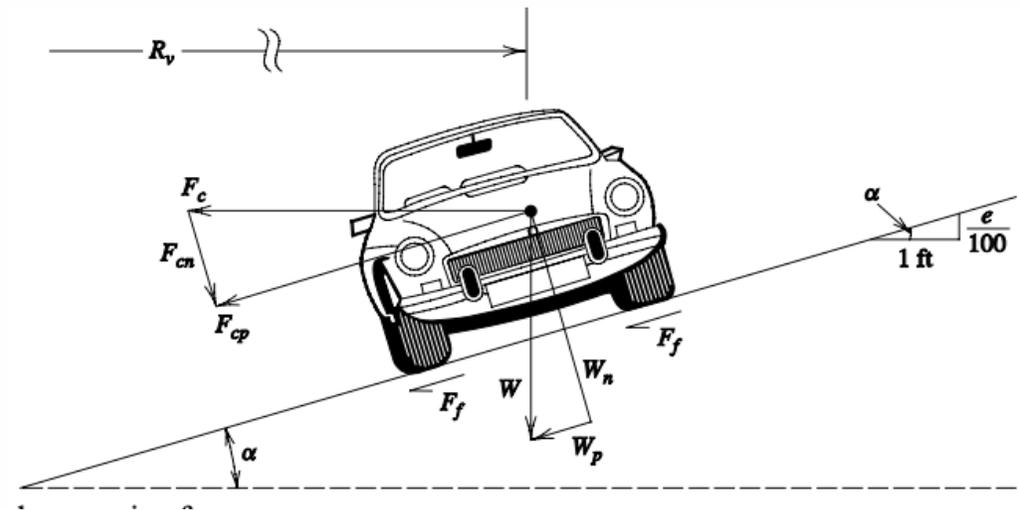
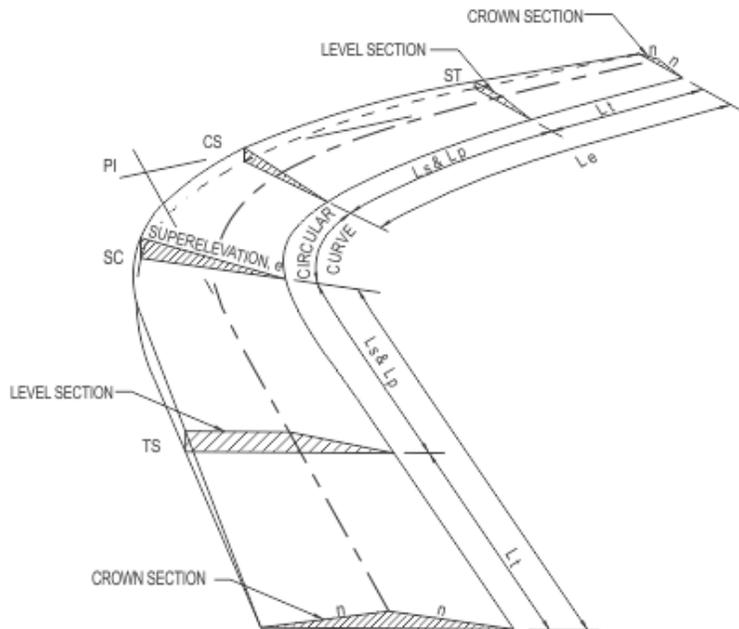
$$K = \frac{L}{A} = \frac{1270}{|-4-3|} = 181.4$$

$$L = \frac{A \times SSD^2}{800(H_c - 5)}$$

$$\begin{aligned} H_c &= \frac{A \times SSD^2}{800L} + 5 \\ &= \frac{7 \times 730^2}{800(1270)} + 5 \\ &= 8.67 \text{ ft} \end{aligned}$$

HORIZONTAL ALIGNMENT

- A horizontal curve provides a transition between two straight (or tangent) sections of roadway. A key concern in this directional transition is the ability of the vehicle to negotiate a horizontal curve.
- The highway engineer must design a horizontal alignment to accommodate the cornering capabilities of a variety of vehicles, ranging from nimble sports cars to ponderous trucks
- In connecting straight (tangent) sections of roadway with a horizontal curve, **Vehicle cornering** determines the length of curve that are adequate to provide save transition of the vehicle

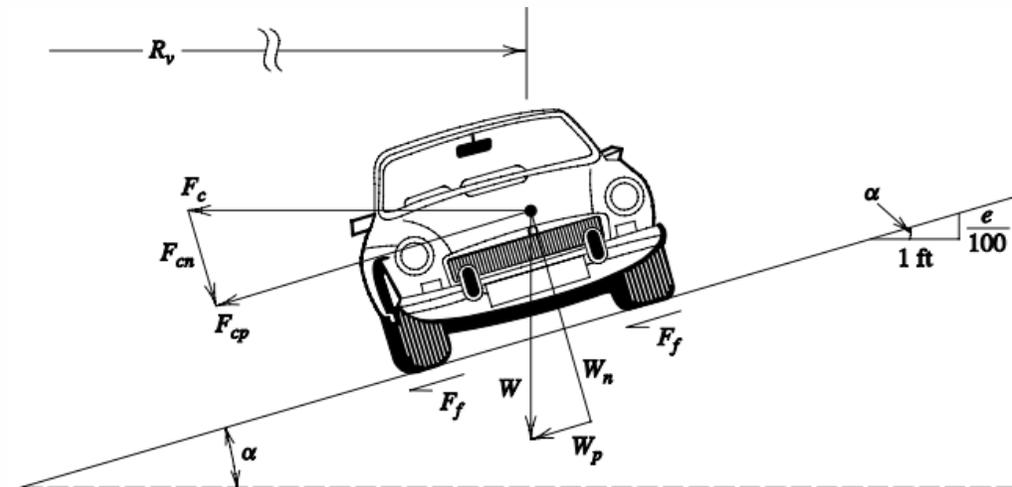


VEHICLE CORNERING

- Vehicle cornering performance is viewed only at the practical design-oriented level, with equations simplified in a manner similar to that used for the stopping-distance equation
- From basic physics, using $W_p + F_f = F_{cp}$ & $F_f = f_s(W_n + F_{cn})$, radius (the vehicle's traveled path in ft) equation can be written as

$$R_v = \frac{V^2}{g \left(f_s + \frac{e}{100} \right)}$$

- The term $\tan \alpha$ indicates the superelevation of the curve (banking) and can be expressed in percent; it is denoted e ($e = 100 \tan \alpha$). In words, the superelevation is the number of vertical feet (meters) of rise per 100 feet (meters) of horizontal distance
- In the actual design of a horizontal curve, the engineer must select appropriate values of e and f_s .
- The value selected for superelevation, e , is critical because high rates of superelevation can cause **vehicle steering problems** on the horizontal curve, and in cold climates, ice on the roadway can **reduce** f_s such that vehicles traveling at less than the design speed on an excessively superelevated curve could **slide inward off the curve** due to gravitational forces.



EXAMPLE 3.14

A roadway is being designed for a speed of **70 mi/h**. At one horizontal curve, it is known that the superelevation is **8.0%** and the coefficient of side friction is **0.10**. **Determine the minimum radius of curve** (measured to the traveled path) that will provide for safe vehicle operation.

SOLUTION

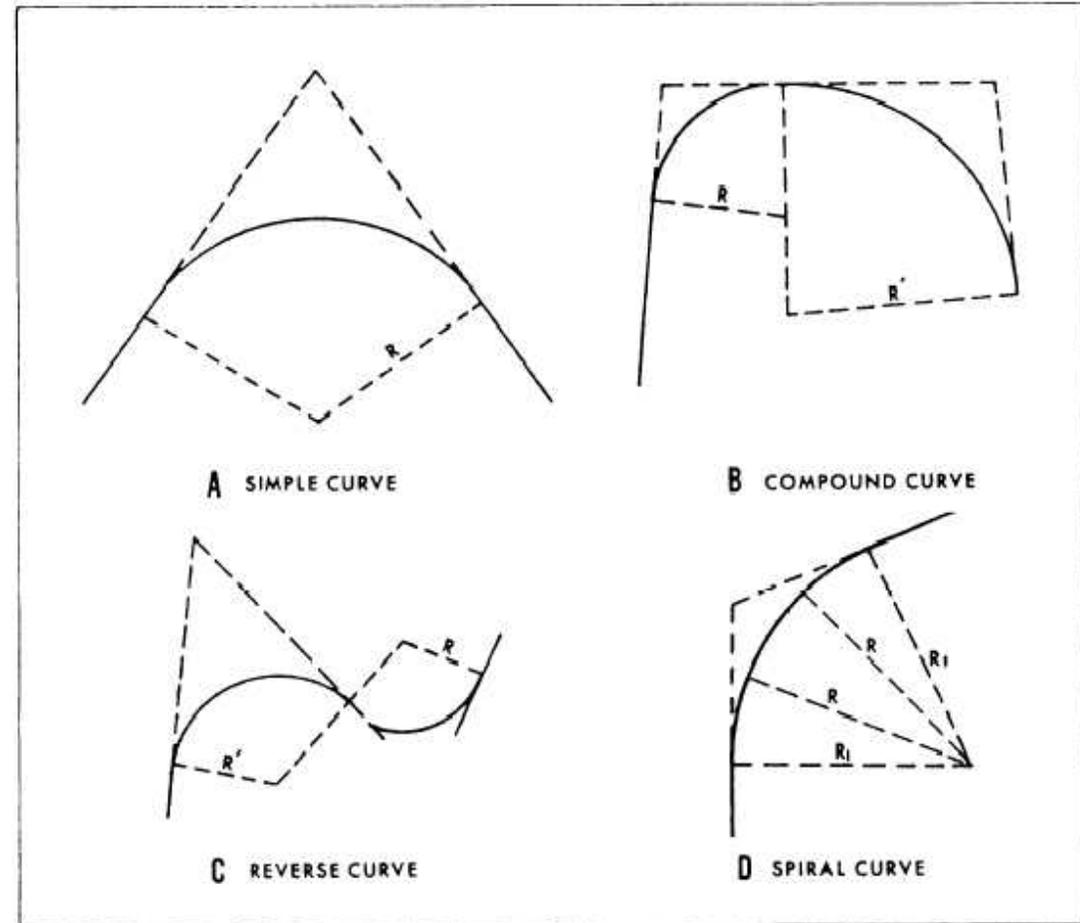
- [with $1.467 = (5280/3600)$] converting mi/h to ft/s

$$R_v = \frac{V^2}{g \left(f_s + \frac{e}{100} \right)} = \frac{(70 \times 1.467)^2}{32.2(0.10 + 0.08)} = \underline{\underline{1819.40 \text{ ft}}}$$

- This value is the minimum radius, because radii smaller than 1819.40 ft will generate centripetal forces higher than those that can be safely supported by the superelevation and the side frictional force.

HORIZONTAL CURVE FUNDAMENTALS

- In connecting straight (tangent) sections of roadway with a horizontal curve, several options are available.
 - The most obvious of these is the simple **circular curve**, which is just a curve with a single, constant radius.
 - **Reverse curves** generally consist of two consecutive curves that turn in opposite directions. They are used to shift the alignment of a highway laterally. Not recommended because drivers may find it difficult to stay within their lane as a result of sudden changes to the alignment.,
 - **Compound curves** consist of two or more curves, usually circular, in succession. used to fit horizontal curves to very specific alignment needs, such as interchange ramps, intersection curves, or difficult topography. it might be difficult for drivers to maintain their lane position as they transition from one curve to the next.
 - **Spiral curves** are curves with a continuously changing radius. it used to transition a tangent section of roadway to a circular curve. spiral curves are not often used. However, Spiral curves are sometimes used on high-speed roadways with sharp horizontal curves and also to gradually introduce the superelevation of an upcoming horizontal curve.



HORIZONTAL CURVE FUNDAMENTALS

Horizontal curve equations

- **the degree of curve**, which is defined as the angle subtended by a 100-ft arc along the horizontal curve. It is a measure of the sharpness of the curve and is frequently used instead of the radius in the construction of the curve.

$$D = \frac{100 \left(\frac{180}{\pi} \right)}{R} = \frac{18000}{\pi R}$$

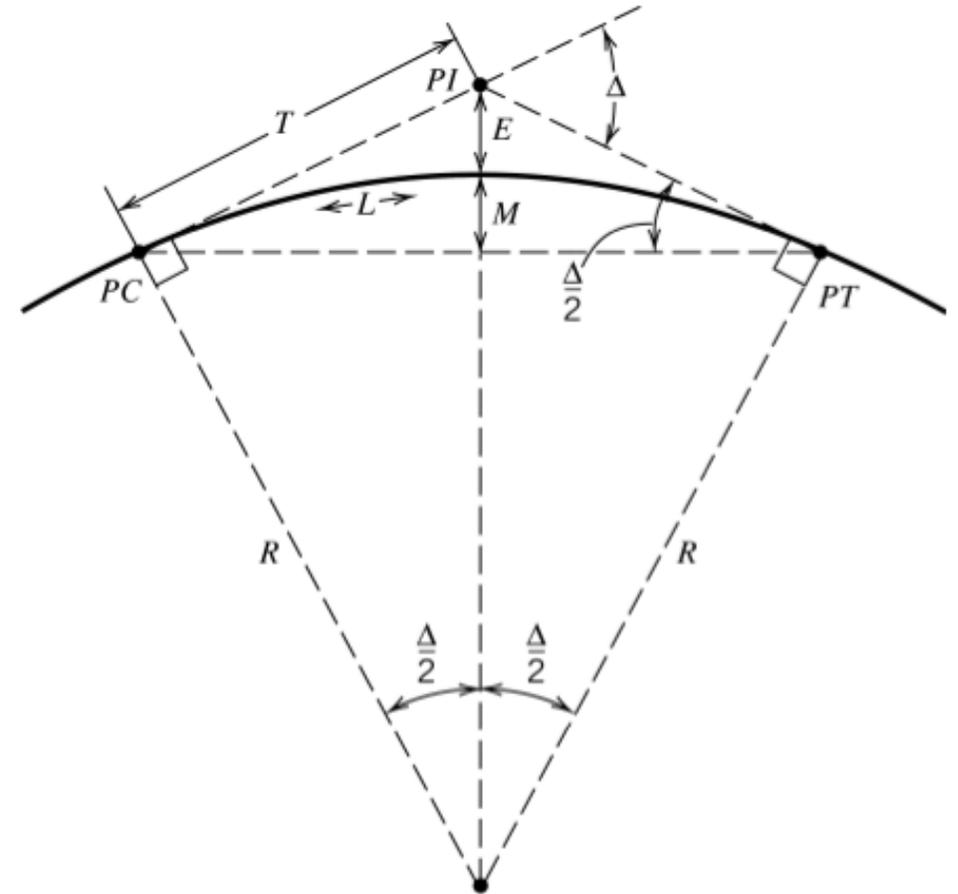
- Geometric and trigonometric analyses of Fig. 3.13 reveal the following relationships:

$$E = R \left(\frac{1}{\cos(\Delta/2)} - 1 \right)$$

$$T = R \tan \frac{\Delta}{2}$$

$$M = R \left(1 - \cos \frac{\Delta}{2} \right)$$

$$L = \frac{\pi}{180} R \Delta$$



It is important to note that horizontal curve stationing, curve length (L), and curve radius (R) are usually measured to the centerline of the road.

EXAMPLE 3.15

A horizontal curve is designed with a **2000-ft radius**. The curve has a tangent length of **400 ft** and the **PI** is at station **103 + 00**. **Determine the stationing of the PT.**

SOLUTION

- determine the central angle, :

$$T = R \tan \frac{\Delta}{2}$$
$$400 = 2000 \tan \frac{\Delta}{2}$$
$$\Delta = 22.62^\circ$$

- the length of the curve is

$$L = \frac{\pi}{180} R \Delta$$
$$L = \frac{3.1416}{180} 2000(22.62) = 789.58 \text{ ft}$$

- Given that the tangent length is 400 ft

$$\text{station of } PC = 103 + 00 \text{ minus } 4 + 00 = 99 + 00$$

Since horizontal curve stationing is measured along the alignment of the road,

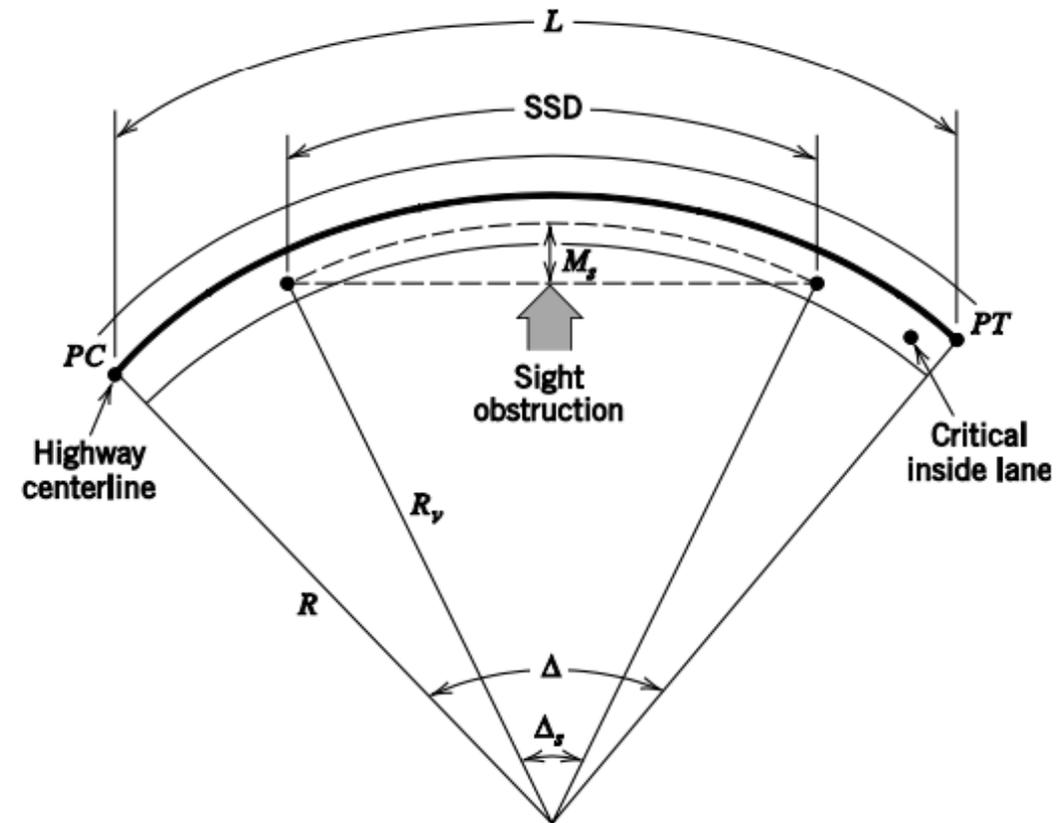
$$\begin{aligned} \text{station of } PT &= \text{station of } PC + L \\ &= 99 + 00 \text{ plus } 7 + 89.58 = 106 + 89.58 \end{aligned}$$

STOPPING SIGHT DISTANCE AND HORIZONTAL CURVE DESIGN

- Once you have a radius that seems to connect the two previously disjointed sections of roadway safely and comfortably, you need to make sure that you have provided an adequate stopping sight distance throughout your horizontal curve.
- Sight distance can be the controlling aspect of horizontal curve design where obstructions are present near the inside of the curve.
- Because the sight obstructions for each curve will be different, no general method for calculating the sight distance has been developed. If you do have a specific obstruction in mind, however, there is an equation that might be helpful. This equation involves the stopping sight distance, the degree of the curve, and the location of the obstruction.

$$L = \frac{\pi}{180} R \Delta \quad \longrightarrow \quad SSD = \frac{\pi}{180} R_v \Delta_s$$

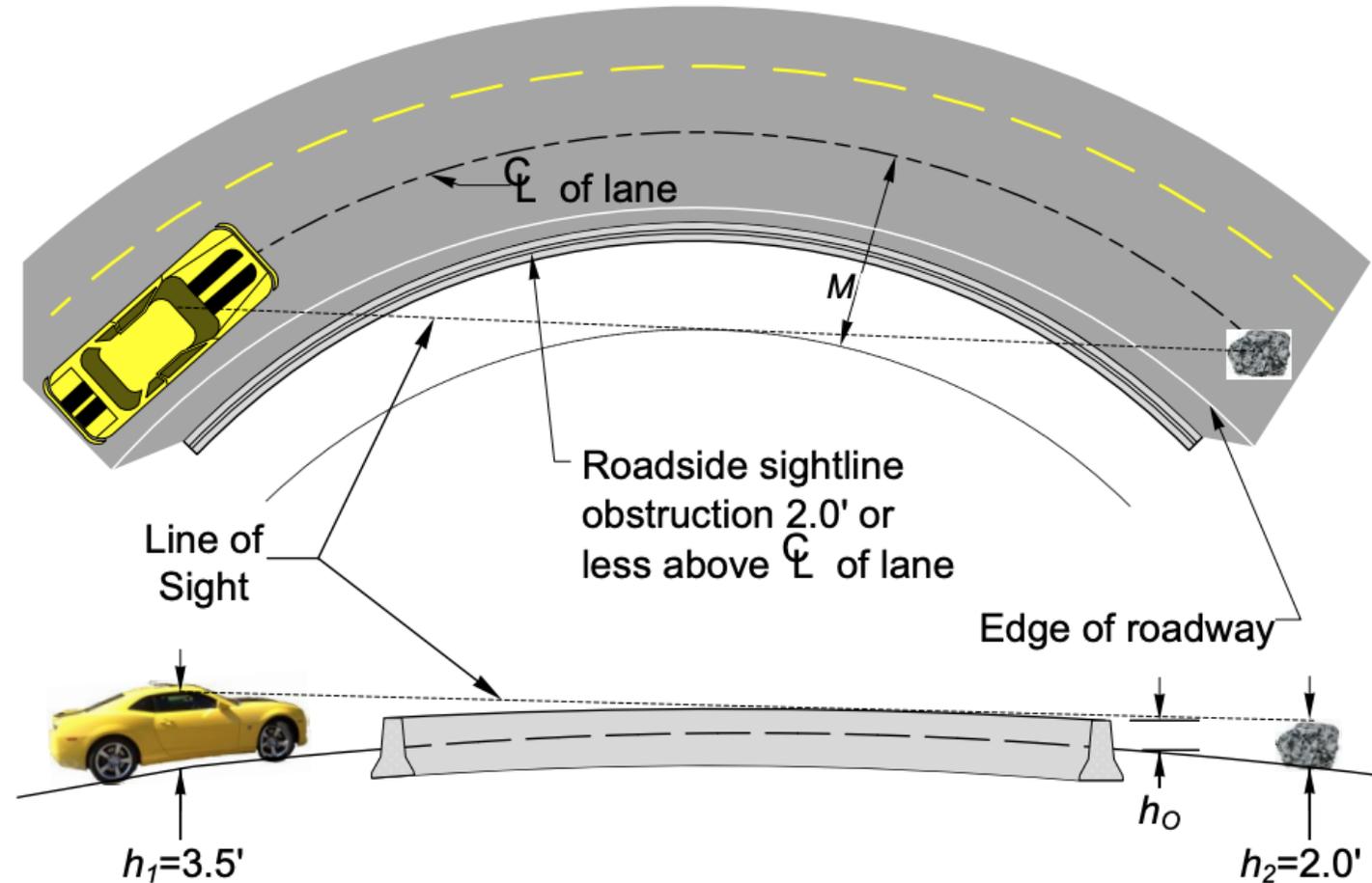
$$M = R \left(1 - \cos \frac{\Delta}{2} \right) \quad \longrightarrow \quad M_s = R_v \left(1 - \cos \frac{90 SSD}{\pi R_v} \right)$$



M_s = Distance from the center of the inside lane to the obstruction (ft.)

STOPPING SIGHT DISTANCE AND HORIZONTAL CURVE DESIGN

- A sightline obstruction is any roadside object within the horizontal sightline offset (M) distance, 2.0 feet or greater above the roadway surface at the centerline of the lane on the inside of the curve.
- If a divided highway has median barrier, the horizontal stopping sight distance for the inside lane of the opposite direction should also be checked and shoulder widening considered.
- The line of sight is assumed to intercept the obstruction at the midpoint of the sight line and 2 feet above the center of the inside lane.



EXAMPLE

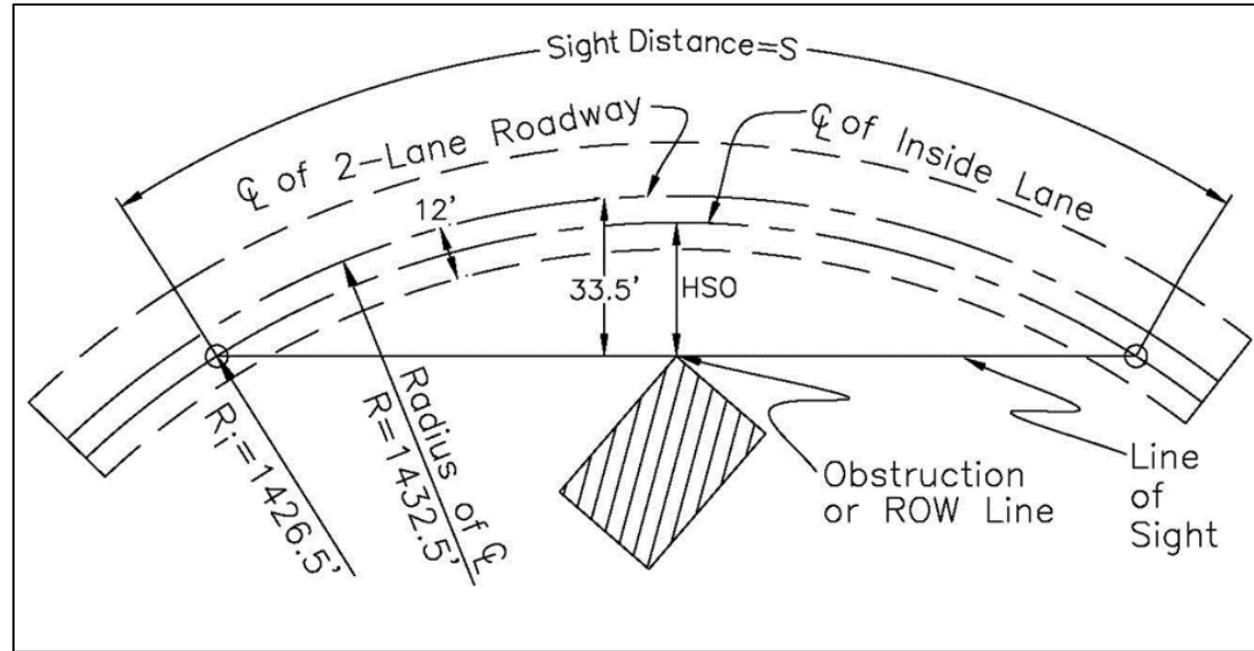


Figure 3-3 Example of Horizontal Stopping Sight Distance on a Two-Lane Roadway

$$R_i = 1432.5 - \frac{12}{2} = 1426.5 \text{ ft}$$

$$HSO = 33.5 - \frac{12}{2} = 27.5 \text{ ft}$$

$$\therefore S = \frac{1426.5}{28.65} \arccos\left(\frac{1426.5 - 27.5}{1426.5}\right) = 561 \text{ ft}$$

EXAMPLE

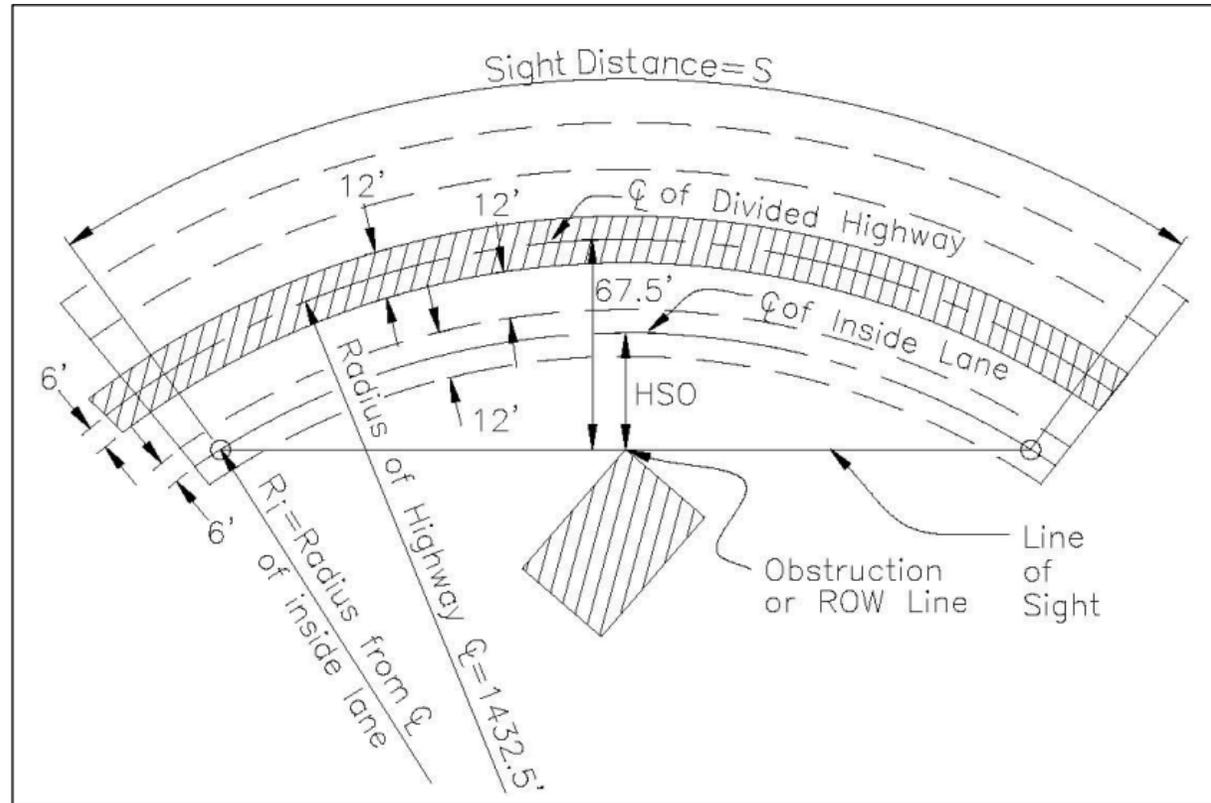


Figure 3-5 Example of Horizontal Stopping Sight Distance on a Divided Highway

$$R_i = 1432.5 - \frac{12}{2} - 12 - \frac{12}{2} = 1408.5 \text{ ft}$$

$$HSO = 67.5 - \frac{12}{2} - 12 - \frac{12}{2} = 43.5 \text{ ft}$$

$$\therefore S = \frac{1408.5}{28.65} \arccos\left(\frac{1408.5 - 43.5}{1408.5}\right) = 702 \text{ ft}$$

EXAMPLE 3.16

A horizontal curve on a **4-lane highway** (two lanes each direction with no median) has a **superelevation of 6%** and a central angle of **40 degrees**. The PT of the curve is at station **322 + 50** and the PI is at **320 + 08**. The road has **10-ft lanes and 8-ft shoulders** on both sides with high retaining walls going up immediately next to the shoulders. **What is the highest safe speed of this curve (highest in 5 mi/h increments) and what is the station of the PC?**

SOLUTION

$$R = \frac{T}{\tan(\Delta/2)} \text{ and, } R = \frac{L}{\Delta(\pi/180)} \implies \frac{T}{\tan(\Delta/2)} = \frac{L}{\Delta(\pi/180)}$$

$$T = PI - PC = (320 + 08) - PC$$

$$L = PT - PC = (322 + 50) - PC$$

$$\frac{(320 + 08) - PC}{\tan(40/2)} = \frac{(322 + 50) - PC}{40(\pi/180)} \implies PC = 317 + 44.25$$

$$T = 32008 - 31744.25 = 263.75 \text{ ft}$$

EXAMPLE 3.16

A horizontal curve on a **4-lane highway** (two lanes each direction with no median) has a **superelevation of 6%** and a central angle of **40 degrees**. The PT of the curve is at station **322 + 50** and the PI is at **320 + 08**. The road has **10-ft lanes and 8-ft shoulders** on both sides with high retaining walls going up immediately next to the shoulders. **What is the highest safe speed of this curve (highest in 5 mi/h increments) and what is the station of the PC?**

SOLUTION

$$R = \frac{T}{\tan(\Delta/2)} = \frac{263.75}{\tan(40/2)} = 724.59 \text{ ft}$$

- $R_v = R - 10 - (10/2) = 724.59 - 15 = 709.59 \text{ ft}$. (Because the curve radius is usually taken to the centerline of the roadway and there are two 10-ft lanes before the centerline)
- From Table 3.5 with a superelevation of 6%,
 - at 45 mi/h a radius of 643 ft is needed
 - at 50 mi/h a radius of 833 ft is needed.

Therefore the highest design speed for centripetal force is 45 mi/h (since $709.59 > 643$, the design is acceptable for 45 mi/h because more than the needed radius is available)

Design speed (mi/h)	Maximum e (%)	Limiting values of f_s	Total $(e/100 + f_s)$	Calculated radius, R_v (ft)	Rounded radius, R_v (ft)
10	4.0	0.38	0.42	15.9	16
15	4.0	0.32	0.36	41.7	42
20	4.0	0.27	0.32	86.0	86
25	4.0	0.23	0.27	154.3	154
30	4.0	0.20	0.24	250.0	250
35	4.0	0.18	0.22	371.2	371
40	4.0	0.16	0.20	533.3	533
45	4.0	0.15	0.19	710.5	711
50	4.0	0.14	0.18	925.9	926
55	4.0	0.13	0.17	1186.3	1190
60	4.0	0.12	0.16	1500.0	1500
10	6.0	0.38	0.44	15.2	15
15	6.0	0.32	0.38	39.5	39
20	6.0	0.27	0.33	80.8	81
25	6.0	0.23	0.29	143.7	144
30	6.0	0.20	0.26	230.8	231
35	6.0	0.18	0.24	340.3	340
40	6.0	0.16	0.22	484.8	485
45	6.0	0.15	0.21	642.9	643
50	6.0	0.14	0.20	833.3	833
55	6.0	0.13	0.19	1061.4	1060
60	6.0	0.12	0.18	1333.3	1330
65	6.0	0.11	0.17	1656.9	1660
70	6.0	0.10	0.16	2041.7	2040
75	6.0	0.09	0.15	2500.0	2500
80	6.0	0.08	0.14	3047.6	3050

EXAMPLE 3.16

- check for adequate sight distance,
 - M_s = shoulder width + half of the inside lane width = $8 + 10/2 = 13$ ft.
- Consider the stopping sight distance required,
 - at 40 mi/h the required stopping sight distance (SSD) is 305 ft

$$M_s = R_v(1 - [\cos \frac{(900)SSD}{\pi(R_v)}]) = (709.59)(1 - [\cos \frac{(900)(305)}{\pi(709.59)}]) = 16.34 ft$$

- Because 16.34 ft is greater than the 13 ft of available M_s , **40 mi/h is too fast.**
- Consider a speed of 35 mi/h which gives SSD = 250 ft

$$M_s = R_v(1 - [\cos \frac{(90)SSD}{\pi(R_v)}]) = (709.59)(1 - [\cos \frac{(90)(250)}{\pi(709.59)}]) = 10.99 ft$$

- Because 10.99 ft is less than 13 ft, **the highway is safe at 35 mi/h.**
- Considering both the maximum safe speeds for centripetal force (45 mi/h) and sight distance (35 mi/h), the lower of the two speeds will govern. Thus **35 mi/h (the highest safe speed for sight distance) is the lower of the two speeds and is the highest safe speed for this curve.**

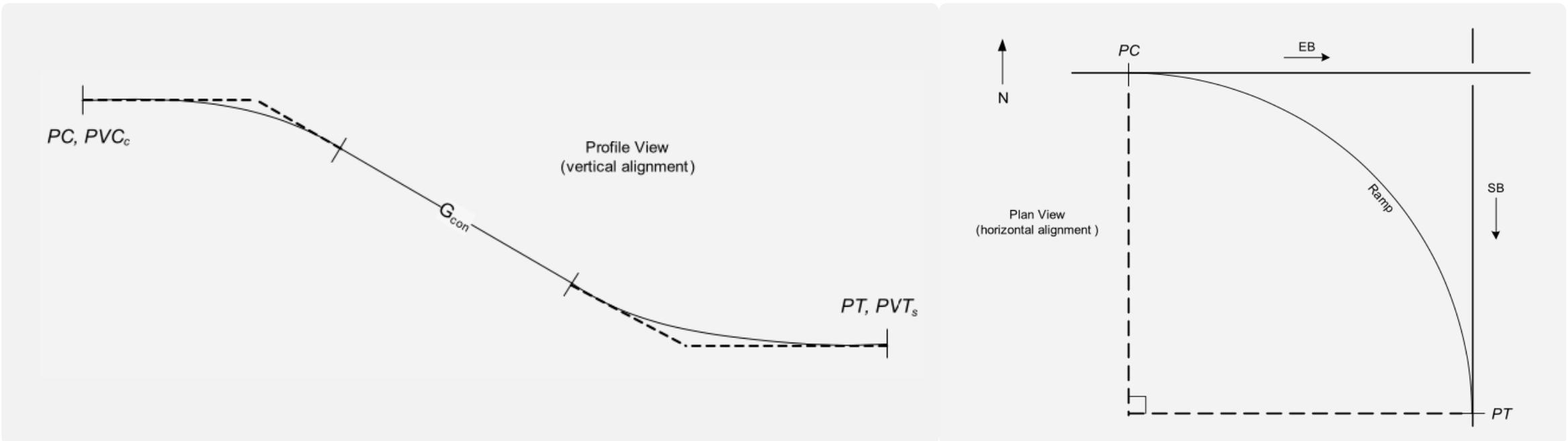
Table 3.1 Stopping Sight Distance

Design speed (mi/h)	Brake reaction distance (ft)	Braking distance on level (ft)	Stopping sight distance	
			Calculated (ft)	Design (ft)
15	55.1	21.6	76.7	80
20	73.5	38.4	111.9	115
25	91.9	60.0	151.9	155
30	110.3	86.4	196.7	200
35	128.6	117.6	246.2	250
40	147.0	153.6	300.6	305
45	165.4	194.4	359.8	360
50	183.8	240.0	423.8	425
55	202.1	290.3	492.4	495
60	220.5	345.5	566.0	570
65	238.9	405.5	644.4	645
70	257.3	470.3	727.6	730
75	275.6	539.9	815.5	820
80	294.0	614.3	908.3	910

EXAMPLE 3.18:

DESIGN OF A COMBINED HORIZONTAL/VERTICAL ALIGNMENT

A new highway is to be constructed over an existing highway. The two highways will **intersect at right angles** and are to be grade-separated. Both highways are **level grade** (constant elevation). The new highway will run east-west and the existing highway runs north-south at **elevation of 565.5 ft**. The proposed bridge structure for the new highway is such that the **bridge girder thickness is 6 ft** (measured from the road surface to the bottom of the girder). A **single-lane ramp** is to be constructed to allow eastbound traffic to go southbound. A **single horizontal curve, with a central angle of 90 degrees**, is to be used. With a **design speed of 40 mi/h** and a required **superelevation of 4%**, **determine the following: the stationing of the PC, PI, and PT**, assuming the curve begins at station 40 + 00;



EXAMPLE 3.18: DESIGN OF A COMBINED HORIZONTAL/VERTICAL ALIGNMENT

a) Horizontal Design

- From Table 3.5 with a 40 mi/h design speed and 4% superelevation,
 - $R_v = 533$ ft (Because the ramp is a single lane, $R = R_v$)
- The length of the horizontal curve as (with $R = 533$ ft and $\Delta = 90$ degrees),

$$L = R\Delta \frac{\pi}{180} = (533)(90) \frac{3.1416}{180} = 837.24 \text{ ft}$$

- The tangent length $T = R = 533$ ft. (Because it's single horizontal curve, with a central angle of 90 degrees)
 - station of PC = 40+00 ft
 - station of PI = station of PC + T = (40+00) + (5+33) = (45+33) ft
 - station of PT = station of PC+L = (40+00) + (8+37.24) = (48+37.24) ft
- The distance that must be cleared from the inside of the horizontal curve to provide sufficient stopping sight distance (with $R_v = 533$ and SSD = 305 ft (from

Table 3.1 at 40 mi/h))

$$M_s = R_v \left(1 - \left[\cos \frac{(90)SSD}{\pi(R_v)} \right] \right) = (533) \left(1 - \left[\cos \frac{(90)(305)}{\pi(533)} \right] \right) = 21.67 \text{ ft}$$

Thus, a distance of at least **21.67 ft** must be cleared from the center of the ramp's lane to the nearest sight obstruction on the inside of the curve.

Design speed (mi/h)	Maximum e (%)	Limiting values of f_s	Total $(e/100 + f_s)$	Calculated radius, R_v (ft)	Rounded radius, R_v (ft)
10	4.0	0.38	0.42	15.9	16
15	4.0	0.32	0.36	41.7	42
20	4.0	0.27	0.32	86.0	86
25	4.0	0.23	0.27	154.3	154
30	4.0	0.20	0.24	250.0	250
35	4.0	0.18	0.22	371.2	371
40	4.0	0.16	0.20	533.3	533
45	4.0	0.15	0.19	710.5	711
50	4.0	0.14	0.18	925.9	926
55	4.0	0.13	0.17	1186.3	1190
60	4.0	0.12	0.16	1500.0	1500

Design speed (mi/h)	Brake reaction distance (ft)	Braking distance on level (ft)	Stopping sight distance	
			Calculated (ft)	Design (ft)
15	55.1	21.6	76.7	80
20	73.5	38.4	111.9	115
25	91.9	60.0	151.9	155
30	110.3	86.4	196.7	200
35	128.6	117.6	246.2	250
40	147.0	153.6	300.6	305
45	165.4	194.4	359.8	360
50	183.8	240.0	423.8	425
55	202.1	290.3	492.4	495
60	220.5	345.5	566.0	570
65	238.9	405.5	644.4	645
70	257.3	470.3	727.6	730
75	275.6	539.9	815.5	820
80	294.0	614.3	908.3	910

EXAMPLE 3.18:

DESIGN OF A COMBINED HORIZONTAL/VERTICAL ALIGNMENT

b) Vertical Design

- $K_c = 44$, and $K_s = 64$ (for 40 mi/h,)
- Adequate clearance must be provided over the existing highway.
 - AASHTO [2011] specifies a desirable clearance height of 16.5 ft.
 - The bridge girder thickness is given as 6 ft
 - the total elevation difference between the two highways is **22.5 ft (16.5 + 6)**
- The elevation change will be (the final offsets of the sag and crest curves) + (the change in elevation resulting from the constant-grade section connecting the two curves).
 - $G_{2s} = G_{1c} = G_{con}$, and since $G_{1s} = G_{2c} = 0$ (as in Example 3.8), $G_{con} = A_s = A_c = A$.
 - the available distance of 837.24 ft (known from the length of the horizontal curve)

$$\frac{AL_s}{200} + \frac{AL_c}{200} + \frac{A(837.24 - L_s - L_c)}{100} = 22.5$$

- Using $L = KA$, we have

$$\begin{aligned} \frac{A^2 K_s}{200} + \frac{A^2 K_c}{200} + \frac{A(837.24 - K_s A - K_c A)}{100} &= 22.5 \\ 0.54A^2 + 8.374A - 1.08A^2 &= 22.5 \\ -0.54A^2 + 8.374A - 22.5 &= 0 \end{aligned}$$

Solving this gives $A = 3.458$ and $A = 12.049$; **$A = 3.458\%$** is chosen because we want to minimize the grade.

Design speed (mi/h)	Stopping sight distance (ft)	Rate of vertical curvature, K^*	
		Calculated	Design
15	80	3.0	3
20	115	6.1	7
25	155	11.1	12
30	200	18.5	19
35	250	29.0	29
40	305	43.1	44
45	360	60.1	61
50	425	83.7	84
55	495	113.5	114
60	570	150.6	151
65	645	192.8	193
70	730	246.9	247
75	820	311.6	312
80	910	383.7	384

Table 3.3 Design Controls for Sag Vertical Curves Based on Stopping Sight Distance

Design speed (mi/h)	Stopping sight distance (ft)	Rate of vertical curvature, K^*	
		Calculated	Design
15	80	9.4	10
20	115	16.5	17
25	155	25.5	26
30	200	36.4	37
35	250	49.0	49
40	305	63.4	64
45	360	78.1	79
50	425	95.7	96
55	495	114.9	115
60	570	135.7	136
65	645	156.5	157
70	730	180.3	181
75	820	205.6	206
80	910	231.0	231

EXAMPLE 3.18:

DESIGN OF A COMBINED HORIZONTAL/VERTICAL ALIGNMENT

- The curve lengths are

$$L_s = K_s A = 64(3.458) = \underline{\underline{221.31 \text{ ft}}}$$

$$L_c = K_c A = 44(3.458) = \underline{\underline{152.15 \text{ ft}}}$$

$$L_{\text{con}} = 837.24 - 221.31 - 152.15 = 463.78 \text{ ft}$$

- The stationing and elevation of the key points along the vertical alignment can now be calculated,
 - The elevation of the new east-west road will be 588 ft which is determined by (22.5 ft + 565.5 ft).
 - The PC and PVC_c are both at station 40 + 00 and elevation 588 ft

$$\begin{aligned} \text{station of } PVC_c &= \text{station of } PC \\ &= \underline{\underline{40+00}} \\ \text{station of } PVI_c &= 40+00 + L_c / 2 = 40+00 \text{ plus } (1+ 52.15 / 2) = \underline{\underline{41+52.15}} \\ \text{station of } PVT_c &= \text{station of } PVI_c + L_c \\ &= 40+00 \text{ plus } (1+ 52.15) = \underline{\underline{41+52.15}} \\ \text{station of } PVC_s &= \text{station of } PVT_c + L_{\text{con}} \\ &= 41+52.15 \text{ plus } 4+ 63.78 = \underline{\underline{46+15.93}} \\ \text{station of } PVI_s &= \text{station of } PVC_s + L_s / 2 \\ &= 46+15.93 \text{ plus } (2+ 21.31) / 2 = \underline{\underline{47+26.59}} \\ \text{station of } PVT_s &= \text{station of } PT \\ &= \text{station of } PVC_s + L_s \\ &= 46+15.93 \text{ plus } (2+ 21.31) = \underline{\underline{48+37.24}} \end{aligned}$$

$$\begin{aligned} \text{elevation } PVT_c &= \text{elev } PVI_c - \frac{AL_c}{200} \\ &= 588 - \frac{3.458(152.15)}{200} = \underline{\underline{585.37 \text{ ft}}} \\ \text{elevation } PVC_s &= \text{elev } PVT_c - \frac{AL_{\text{con}}}{100} \\ &= 585.37 - \frac{3.458(463.78)}{100} = \underline{\underline{569.33 \text{ ft}}} \\ \text{elevation } PVT_s &= \text{elev } PVC_s - \frac{AL_c}{200} \\ &= 569.33 - \frac{3.458(221.31)}{200} = \underline{\underline{565.00 \text{ ft}}} \end{aligned}$$