ROAD VEHICLE PERFORMANCE

The objective of this topic is to introduce the basic principles of road vehicle performance. Primary attention will be given to the straight-line performance of vehicles (**acceleration, deceleration, top speed, and the ability to ascend grades**).

INTRODUCTION

The performance of road vehicles forms the basis for highway design guidelines and traffic analysis to serves two important functions:

- It provides insight into highway design and traffic operations and the **compromises** that are necessary to accommodate the wide variety of vehicles.
- It forms a basis from which the **impact** of advancing vehicle technologies on existing highway design guidelines can be assessed.

INTRODUCTION

The performance of road vehicles forms the basis for highway design guidelines and traffic analysis

1 Highway design

Example:

-The determination of the length of freeway acceleration and deceleration lanes

-Maximum highway grades

-Stopping sight distances & passing sight distances.

2 Traffic analysis

Example:

-The selection and design of traffic control devices

-The determination of speed limits

-The timing and control of traffic signal systems.

TRACTIVE EFFORT AND RESISTANCE

Tractive effort (also referred to as thrust) and resistance are the two primary opposing forces that determine the straight-line performance of road vehicles.

- **Tractive effort** is simply the force available, at the roadway surface, to perform work and is expressed in lb (N).
- **Resistance**, also expressed in lb (N), is defined as the force impeding vehicle motion.

The three major sources of vehicle resistance are:

1. Aerodynamic resistance
2. Rolling resistance (which originates from the roadway surface–tire interface)
3. Grade or gravitational resistance

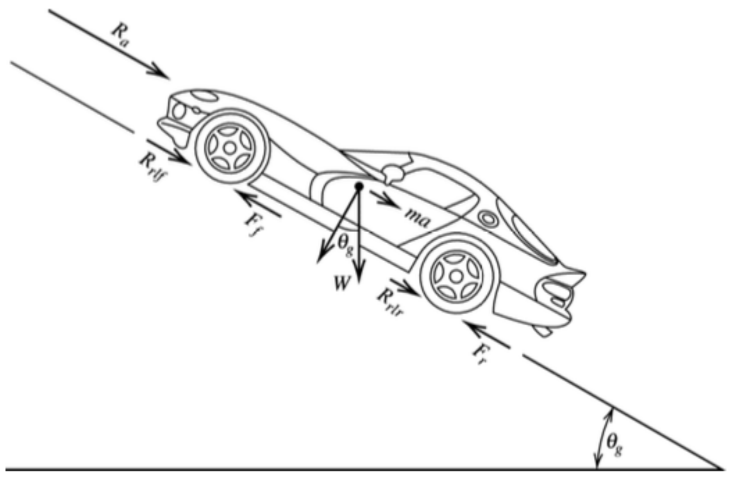


Figure 2.1 Forces acting on a road vehicle.

R_a = aerodynamic resistance in lb,
R_{rlf} = rolling resistance of the front tires in lb,
R_{rlr} = rolling resistance of the rear tires in lb,
F_f = available tractive effort of the front tires in lb,
F_r = available tractive effort of the rear tires in lb,
W = total vehicle weight in lb,
θ_g = angle of the grade in degrees,
m = vehicle mass in slugs, and
a = acceleration in ft/s^2.

TRACTIVE EFFORT AND RESISTANCE

$$F_f + F_r = ma + R_a + R_{rlf} + R_{rlr} + W sin\theta_g$$

$$F_f + F_r = ma + R_a + R_{rlf} + R_{rlr} + R_g$$

$$F = ma + R_a + R_r + R_g$$

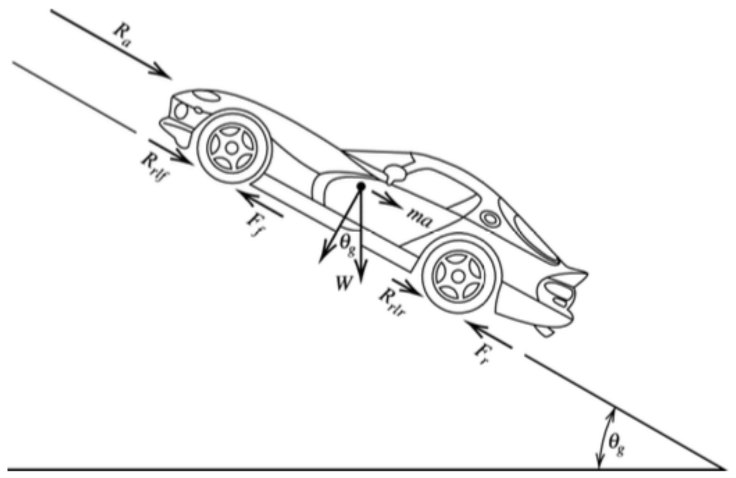


Figure 2.1 Forces acting on a road vehicle.

R_a = aerodynamic resistance in lb,
R_{rlf} = rolling resistance of the front tires in lb,
R_{rlr} = rolling resistance of the rear tires in lb,
F_f = available tractive effort of the front tires in lb,
F_r = available tractive effort of the rear tires in lb,
W = total vehicle weight in lb,
θ_g = angle of the grade in degrees,
m = vehicle mass in slugs, and
a = acceleration in ft/s^2.

1. AERODYNAMIC RESISTANCE

Aerodynamic resistance is a resistive force that can have significant impacts on vehicle performance.

Aerodynamic efficiency in design has long been the rule in racing and sports cars for such a concern over fuel efficiency
and overall vehicle performance

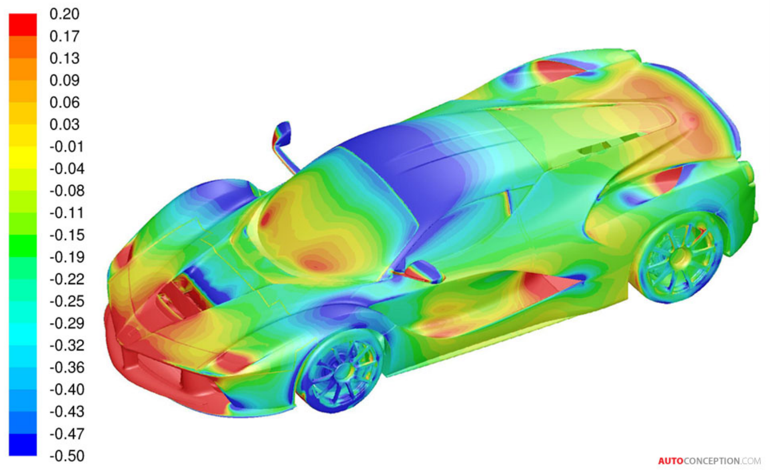
Aerodynamic resistance originates from three **sources** (C_D):

- The turbulent flow of air around the vehicle body (over 85% of total aerodynamic resistance).
- The friction of the air passing over the body of the vehicle (on the order of 12% of total aerodynamic resistance)
- Air flow through vehicle components such as radiators and air vents (3% of the total aerodynamic resistance)

$$R_a = \frac{\rho}{2} C_D A_f V^2 \qquad \textbf{(2.3)}$$

where

R_a = aerodynamic resistance in lb,
ρ = air density in slugs/ft^3,
C_D = coefficient of drag (unitless),
A_f = frontal area of the vehicle (projected area of the vehicle in the direction of travel) in ft^2, and
V = speed of the vehicle in ft/s.



1. AERODYNAMIC RESISTANCE

$$R_a = \frac{\rho}{2} C_D A_f V^2$$

With power being the product of force and speed, the multiplication of Eq. 2.3 by speed gives

$$P_{R_a} = \frac{\rho}{2} C_D A_f V^3$$

Or

$$hp_{R_a} = \frac{\rho}{1100} C_D A_f V^3$$

Table 2.1 Typical Values of Air Density Under Specified Atmospheric Conditions

Altitude (ft)	Temperature (°F)	Pressure (lb/in^2)	Air density ($slugs/ft^3$)
0	59.0	14.7	0.002378
5,000	41.2	12.2	0.002045
10,000	23.4	10.1	0.001755

Table 2.2 Ranges of Drag Coefficients for Typical Road Vehicles

Vehicle type	Drag coefficient (C_D)
Automobile	0.25–0.55
Bus	0.5–0.7
Tractor-Trailer	0.6–1.3
Motorcycle	0.27–1.8

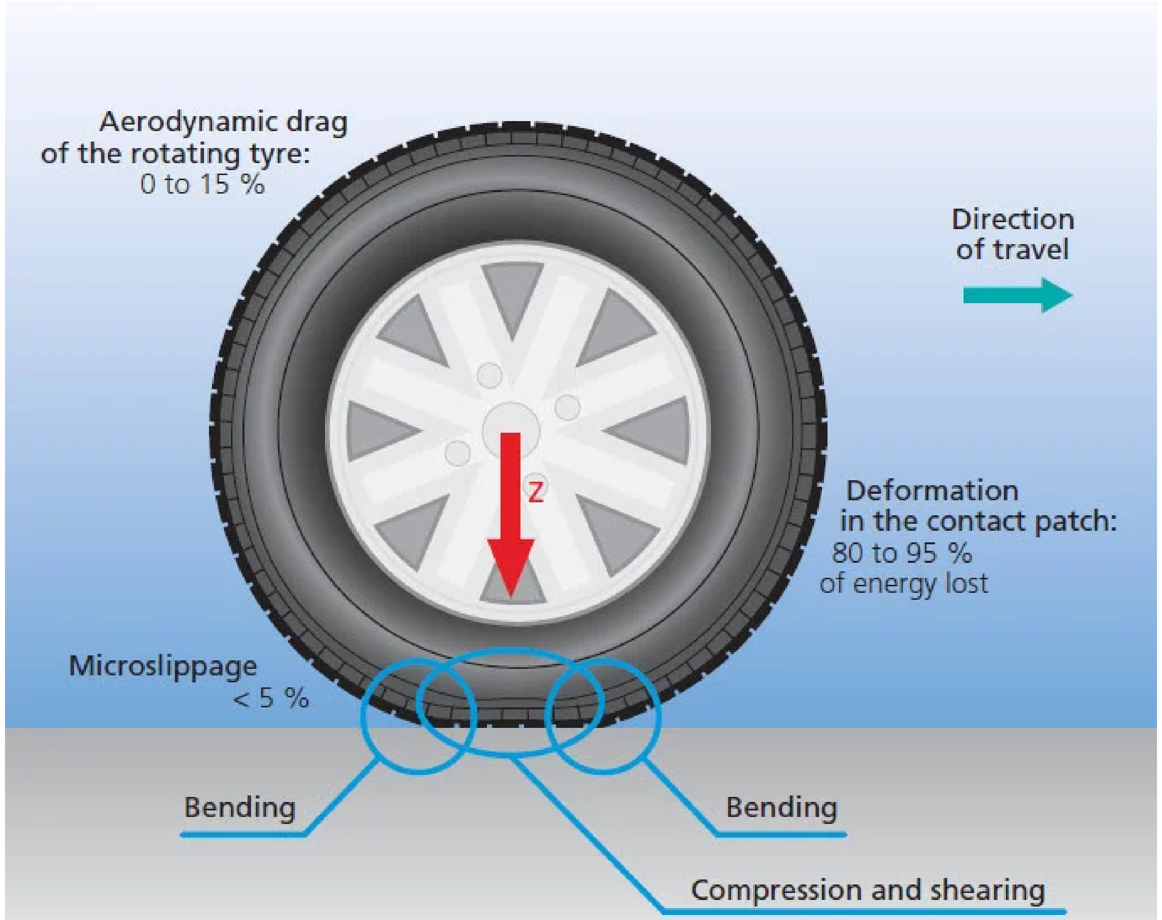
2. ROLLING RESISTANCE

Rolling resistance refers to the resistance generated from a vehicle's internal mechanical friction and from pneumatic tires and their interaction with the roadway surface.

Rolling resistance originates from three **sources** (C_D):

- The deformation of the tire as it passes over the roadway surface (90% of the total rolling resistance).
- Vehicle weights and pavement types, penetration and compression (around 4% of the total rolling resistance).
- Frictional motion due to the slippage of the tire on the roadway surface and, to a lesser extent, air circulation around the tire and wheel (the fanning effect) (roughly 6% of the total rolling resistance

The main three physical causes of rolling resistance



2. ROLLING RESISTANCE

Factors influence rolling resistance:

1. **the rigidity of the tire and the roadway surface** influence the degree of tire penetration, surface compression, and tire deformation:
   -Hard, smooth, and dry roadway surfaces provide the lowest rolling resistance.
2. **tire conditions, including inflation pressure and temperature**
   -High tire inflation decreases rolling resistance on hard paved surfaces as a result of reduced friction but increases rolling resistance on soft unpaved surfaces due to additional surface penetration. Also, higher tire temperatures make the tire body more flexible, and thus less resistance is encountered during tire deformation.
3. **vehicle's operating speed**
   -Increasing speed results in additional tire flexing and vibration and thus a higher rolling resistance.

2. ROLLING RESISTANCE

Overall rolling resistance can be approximated as the product of a friction term (coefficient of rolling resistance) and the weight of the vehicle acting normal to the roadway surface.

The coefficient of rolling resistance for road vehicles operating on paved surfaces is approximated as

$$f_{rl} = 0.01(1 + \frac{V}{44.73})$$

$$f_{rl} = 0.01(1 + \frac{V}{147})$$

f_{rl} = coefficient of rolling resistance (unitless)

V = vehicle speed in ft/s

2. ROLLING RESISTANCE

The rolling resistance will simply be the coefficient of rolling resistance multiplied by ($W \cos \theta_g$)

$$R_{rl} = f_{rl} W cos\theta_g$$

For most highway applications θ_g is quite small, so it can be assumed that $\cos \theta_g = 1$, giving the equation for rolling resistance (R_{rl}) as

$$R_{rl} = f_{rl} W$$

The amount of power required to overcome rolling resistance is

$$P_{R_{rl}} = f_{rl} W V \qquad or \qquad hp_{R_{rl}} = \frac{f_{rl} W V}{550}$$

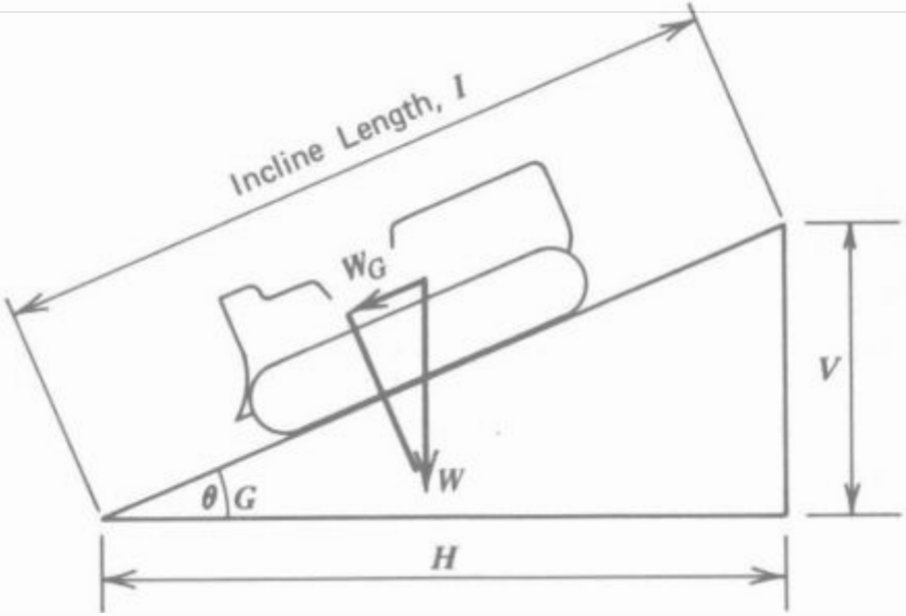
P_{Rrl} = power required to overcome rolling resistance (N-m/s (Watts)

W = total vehicle weight in lb.(N)

3. GRADE RESISTANCE

Grade resistance represents the gravitational force that component of vehicle weight which acts parallel to an inclined surface.

- when the vehicle is traveling up a grade, grade resistance is positive.
- when traveling downhill, grade resistance is negative.



As explained before, the expression for grade resistance (Rg) is

$$R_g = W sin\theta_g$$

Highway grades are usually very small, so we can say:

$$sin\theta_g = \frac{V}{I} \qquad and \qquad tan\theta_g = \frac{V}{H}$$

$$sin\theta_g \cong tan\theta_g = G \qquad then \qquad R_g = W tan\theta_g = WG$$

G = grade in percentage, defined as the vertical rise per some specified horizontal distance in ft/ft.

EXAMPLE 2.1 **AERODYNAMIC AND ROLLING RESISTANCE**

A 2500-lb car is driven at sea level ($\rho = 0.002378$ slugs/ft^3) on a level paved surface. The car has $C_D = 0.38$ and 20 ft^2 of frontal area. It is known that at maximum speed, 50 hp is being expended to overcome rolling and aerodynamic resistance. Determine the car's maximum speed.

SOLUTION

It is known that at maximum speed (V_m),

$$\text{available horsepower} = R_a V_m + R_{rl} V_m$$

or

$$\text{available hp} = \frac{\frac{\rho}{2} C_D A_f V_m^3 + f_{rl} W V_m}{550}$$

Substituting, we have

$$50 = \frac{\frac{0.002378}{2}(0.38)(20)V_m^3 + 0.01\left(1 + \frac{V_m}{147}\right)(2500)V_m}{550}$$

or

$$27{,}500 = 0.00904 V_m^3 + 0.17 V_m^2 + 25 V_m$$

Solving for V_m gives

$$V_m = 133 \text{ ft/s or } 90 \text{ mi/h}$$

EXAMPLE 2.2 GRADE RESISTANCE

A 2000-lb car has $C_D = 0.40$, $A_f = 20$ ft^2, and an available tractive effort of 255 lb. If the car is traveling at an elevation of 5000 ft ($\rho = 0.002045$ slugs/ft^3) on a paved surface at a speed of 70 mi/h, what is the maximum grade that this car could ascend and still maintain the 70-mi/h speed?

SOLUTION

To maintain the speed, the available tractive effort will be exactly equal to the summation of resistances. Thus no tractive effort will remain for vehicle acceleration ($ma = 0$). Therefore, Eq. 2.2 can be written as

$$F = R_a + R_{rl} + R_g$$

For grade resistance (using Eq. 2.9),

$$R_g = WG = 2000G$$

for aerodynamic resistance (using Eq. 2.3),

$$\begin{aligned} R_a &= \frac{\rho}{2} C_D A_f V^2 \\ &= \frac{0.002045}{2}(0.4)(20)(70 \times 5280/3600)^2 \\ &= 86.22 \text{ lb} \end{aligned}$$

and for rolling resistance (using Eq. 2.6),

$$\begin{aligned} R_{rl} &= f_{rl} W \\ &= 0.01\left(1 + \frac{70 \times 5280/3600}{147}\right) \times 2000 \\ &= 33.97 \text{ lb} \end{aligned}$$

Therefore,

$$F = 255 = 86.22 + 33.97 + 2000G$$

$$G = \underline{\underline{0.0674}} \quad \text{or a} \quad 6.74\% \text{ grade}$$

4.TRACTIVE EFFORT

Tractive effort is simply the force available, at the roadway surface, to perform work and is expressed in lb (N).

The tractive effort available to overcome resistance and/or to accelerate the vehicle is determined either by

1. the force generated by the vehicle's engine
2. some maximum value that will be a function of the vehicle's weight distribution and the characteristics of the roadway surface—tire interface.

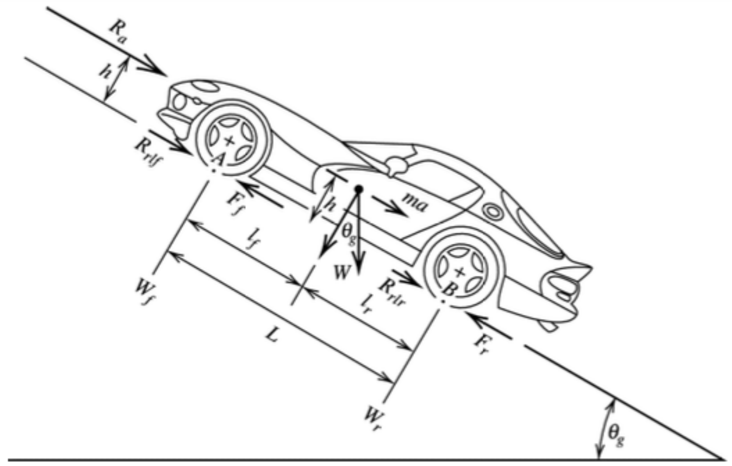


Figure 2.2 Vehicle forces and moment-generating distances.

R_a = aerodynamic resistance in lb,
R_{rlf} = rolling resistance of the front tires in lb,
R_{rlr} = rolling resistance of the rear tires in lb,
F_f = available tractive effort of the front tires in lb,
F_r = available tractive effort of the rear tires in lb,
W = total vehicle weight in lb,
W_f = weight of the vehicle on the front axle in lb,
W_r = weight of the vehicle on the rear axle in lb,
θ_g = angle of the grade in degrees,
m = vehicle mass in slugs,
a = acceleration in ft/s^2,
L = length of wheelbase,
h = height of the center of gravity a roadway surface,
l_f = distance from the front axle to th gravity, and
l_r = distance from the rear axle to th gravity.

4.1. MAXIMUM TRACTIVE EFFORT

No matter how much force a vehicle's engine makes available at the roadway surface, there is a point beyond which additional force merely results in the spinning of tires and does not overcome resistance or accelerate the vehicle.

To determine the maximum tractive effort that the roadway surface–tire contact can support, it is necessary to examine the normal loads on the axles:

- The normal load on the rear axle (Wr) is given by summing the moments about point A while the normal load on the front axle (Wf) is given by summing the moments about point B

$$W_R = \frac{R_a h + W l_f h cos\theta_g + mah \pm W h sin\theta_g}{L}$$

*grade moment (Wh sin θg) is positive for an upward slope and negative for a downward slope.

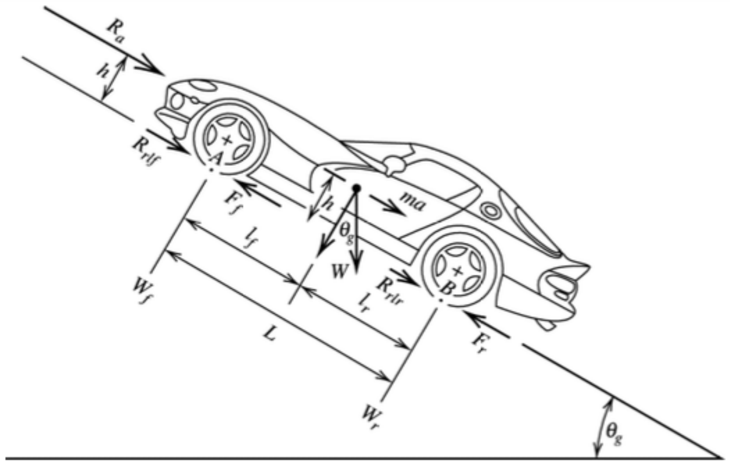


Figure 2.2 Vehicle forces and moment-generating distances.

R_a = aerodynamic resistance in lb,
R_{rlf} = rolling resistance of the front tires in lb,
R_{rlr} = rolling resistance of the rear tires in lb,
F_f = available tractive effort of the front tires in lb,
F_r = available tractive effort of the rear tires in lb,
W = total vehicle weight in lb,
W_f = weight of the vehicle on the front axle in lb,
W_r = weight of the vehicle on the rear axle in lb,
θ_g = angle of the grade in degrees,
m = vehicle mass in slugs,
a = acceleration in ft/s^2,
L = length of wheelbase,
h = height of the center of gravity at roadway surface,
l_f = distance from the front axle to th gravity, and
l_r = distance from the rear axle to the gravity.

4.1. MAXIMUM TRACTIVE EFFORT

$$W_R = \frac{R_a h + W l_f cos\theta_g + mah \pm W h sin\theta_g}{L}$$

Assuming cos θg = 1 for the small grades encountered in highway applications

$$F = ma + R_a + R_r + R_g$$

$$\therefore ma + R_a = F - R_r$$

$$W_R = \frac{W l_f}{L} + \frac{h}{L}(F - R_{rl})$$

the maximum tractive effort as determined by the roadway surface–tire interaction will be the normal force multiplied by the coefficient of road adhesion (μ), so for a rear-wheel–drive car

$$F_{max} = \mu W_R$$

$$\therefore F_{max} = \mu\left[\frac{W l_f}{L} + \frac{h}{L}(F - R_{rl})\right]$$

$$F_{max} = \frac{\mu W\left(l_f - f_{rl} h\right)/L}{1 - \mu h/L} \quad \textbf{(2.14)}$$

Similarly, by summing moments about point *B* (see Fig. 2.2), it can be shown that for a front-wheel–drive vehicle

$$F_{max} = \frac{\mu W\left(l_r + f_{rl} h\right)/L}{1 + \mu h/L} \quad \textbf{(2.15)}$$

4.1. MAXIMUM TRACTIVE EFFORT

EXAMPLE 2.3 **MAXIMUM TRACTIVE EFFORT**

A 2500-lbcar is designed with a 120-inch wheelbase. The center of gravity is located 22 inches above the pavement and 40 inches behind the front axle. If the coefficient of road adhesion is 0.6, what is the maximum tractive effort that can be developed if the car is (a) front-wheel drive and (b) rear-wheel drive?

SOLUTION

For the front-wheel–drive case Eq. 2.15 is used:

$$F_{max} = \frac{\mu W\left(l_r + f_{rl}h\right)/L}{1+\mu h/L}$$

and, from Eq. 2.5, $f_{rl} = 0.01$ because $V = 0$ ft/s, so

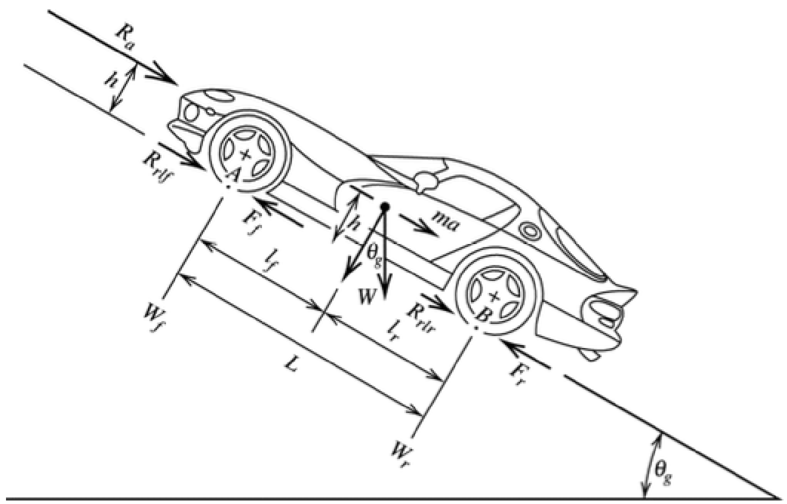
$$F_{max} = \frac{\left[0.6\times 2500\times\left(80+0.01(22)\right)\right]/120}{1+\left(0.6\times 22\right)/120}$$

$$= \underline{\underline{903.38 \text{ lb}}}$$

For the rear-wheel–drive case, Eq. 2.14 is used:

$$F_{max} = \frac{\left[0.6\times 2500\times\left(40-0.01(22)\right)\right]/120}{1-\left(0.6\times 22\right)/120}$$

$$= \underline{\underline{558.71 \text{ lb}}}$$



5. VEHICLE ACCELERATION

For determining vehicle acceleration, the tractive effort (F) and resistance general equation can be applied with an additional term to account for the inertia of the vehicle's rotating parts that must be overcome during acceleration. This term is referred to as **the mass factor (γ_m)**

$$F = \gamma_m ma + R_a + R_r + R_g \qquad \therefore F_{net} = F - \sum R = \gamma_m ma$$

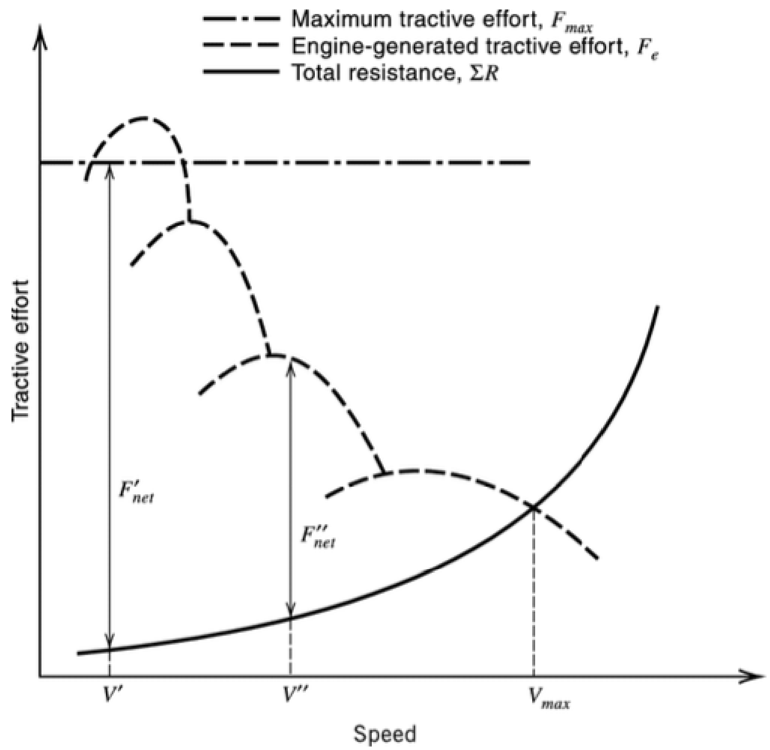
$$\gamma_m = 1.04 + 0.0025\varepsilon_0^2 \qquad \varepsilon_0 : \text{overall gear reduction ratio}$$

Two measures of vehicle acceleration are worthy of note: the **time** to accelerate and the **distance** to accelerate.

$$F_{net} = \gamma_m m(\frac{dV}{dt})$$

$$t = \gamma_m m \int_{V_1}^{V_2} \frac{dV}{f(V)}$$

$$d_a = \gamma_m m \int_{V_1}^{V_2} \frac{VdV}{f(V)}$$



- F_{net} = 0, the vehicle cannot accelerate and is at its maximum speed for specified conditions (grade, air density, engine torque, and so on)

  F_{net} is greater than zero, the vehicle is traveling at a speed less than its maximum speed

EXAMPLE 2.5

A car is traveling at **10 mi/h** on a roadway covered with hard-packed snow (coefficient of road adhesion of **0.20**). The car has **C_D = 0.30, A_f = 20 ft^2, and W = 3000 lb**. The wheelbase is **120 inches**, and the center of gravity is **20 inches** above the roadway surface and **50 inches** behind the front axle. The air density is **0.002045 slugs/ft^3**. The car's engine is producing **95 ft-lb** of torque and is in a gear that gives an overall gear reduction ratio of **4.5 to 1**, while the engine-generated tractive effort is **293.14 lb.** If the driver needs to accelerate quickly to avoid an accident, what would the acceleration be if the car is (a) front-wheel drive and (b) rear-wheel drive?

SOLUTION

We begin by computing the resistances, tractive effort generated by the engine, and mass factor because all of these factors will be the same for both front- and rear-wheel drive.

The aerodynamic resistance is (from Eq. 2.3)

$$
\begin{aligned}
R_a &= \frac{\rho}{2} C_D A_f V^2 \\
&= \frac{0.002045}{2}(0.3)(20)(10\times 5280/3600)^2 \\
&= 1.32 \text{ lb}
\end{aligned}
$$

The rolling resistance is (from Eq. 2.6)

$$
\begin{aligned}
R_{rl} &= f_{rl} W \\
&= 0.01\left(1+\frac{10\times 5280/3600}{147}\right)\times 3000 \\
&= 32.99 \text{ lb}
\end{aligned}
$$

EXAMPLE 2.5

A car is traveling at **10 mi/h** on a roadway covered with hard-packed snow (coefficient of road adhesion of **0.20**). The car has **C_D = 0.30, A_f = 20 ft^2, and W = 3000 lb**. The wheelbase is **120 inches**, and the center of gravity is **20 inches** above the roadway surface and **50 inches** behind the front axle. The air density is **0.002045 slugs/ft^3**. The car's engine is producing **95 ft-lb** of torque and is in a gear that gives an overall gear reduction ratio of **4.5 to 1**, while the engine-generated tractive effort is **293.14 lb.** If the driver needs to accelerate quickly to avoid an accident, what would the acceleration be if the car is (a) front-wheel drive and (b) rear-wheel drive?

The mass factor is (from Eq. 2.20)

$$\begin{aligned}\gamma_m &= 1.04+0.0025\varepsilon_0^2\\ &= 1.04+0.0025(4.5)^2\\ &= 1.091\end{aligned}$$

Recall that, to determine acceleration, we need the resistances (already computed) and the available tractive effort, F, which is the lesser of F_e or F_{max}. For the case of the front-wheel-drive car, Eq. 2.15 can be applied to determine F_{max} :

$$\begin{aligned}F_{\max} &= \frac{\mu W(l_r + f_{rl}h)/L}{1+\mu h/L}\\ &= \frac{\left[0.2\times 3000\times\left(70+0.011(20)\right)\right]/120}{1+(0.2\times 20)/120}\\ &= \underline{\underline{339.77\ \text{lb}}}\end{aligned}$$

Thus for a front-wheel-drive car F = 293.14 lb (the lesser of 293.14 and 339.77) and the acceleration is (from Eq. 2.19)

EXAMPLE 2.5

A car is traveling at **10 mi/h** on a roadway covered with hard-packed snow (coefficient of road adhesion of **0.20**). The car has **C_D = 0.30, A_f = 20 ft^2, and W = 3000 lb**. The wheelbase is **120 inches**, and the center of gravity is **20 inches** above the roadway surface and **50 inches** behind the front axle. The air density is **0.002045 slugs/ft^3**. The car's engine is producing **95 ft-lb** of torque and is in a gear that gives an overall gear reduction ratio of **4.5 to 1**, while the engine-generated tractive effort is **293.14 lb.** If the driver needs to accelerate quickly to avoid an accident, what would the acceleration be if the car is (a) front-wheel drive and (b) rear-wheel drive?

$$F-\sum R=\gamma_m ma$$

$$a=\frac{F-\sum R}{\gamma_m m}=\frac{293.14-34.31}{1.091(3000/32.2)}=\underline{\underline{2.546\ \text{ft/s}^2}}$$

For the case of the rear-wheel-drive car, Eq. 2.14 can be applied to determine F_{max}:

$$F_{max}=\frac{\left[0.2\times3000\times(50-0.011(20))\right]/120}{1-(0.2\times20)/120}=257.48\ \text{lb}$$

Thus for a rear-wheel-drive car F = 257.48 lb (the lesser of 293.14 and 257.48) and the acceleration is (from Eq. 2.19)

$$a=\frac{F-\sum R}{\gamma_m m}=\frac{257.48-34.31}{1.091(3000/32.2)}=\underline{\underline{2.196\ \text{ft/s}^2}}$$

6. FUEL EFFICIENCY AND ACCELERATION

Increase fuel efficiency

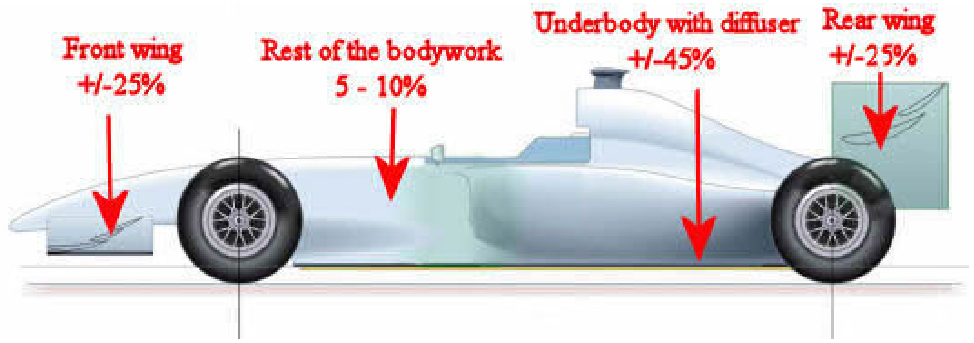
- Decreasing overall vehicle weight (W) will lower grade and rolling resistances, thus reducing fuel consumption (all other factors held constant).
- Aerodynamic improvements such as a lower drag coefficient (C_D) and a reduced frontal area (Af) can produce significant fuel savings.

Improve vehicle acceleration

- Micro-interaction at the tire-pavement interface results in a "cog-type" effect that can increase the coefficient of road adhesion to exceed 1.0 which influences acceleration and braking of a vehicle
- Vehicle aerodynamics (at high speed) can create downward forces that effectively increase Weight of the vehicle which facilitates greater acceleration

Table 2.4 Typical Values of Coefficients of Road Adhesion

	Coefficient of road adhesion	
Pavement	Maximum	Slide
Good, dry	1.00*	0.80
Good, wet	0.90	0.60
Poor, dry	0.80	0.55
Poor, wet	0.60	0.30
Packed snow or ice	0.25	0.10

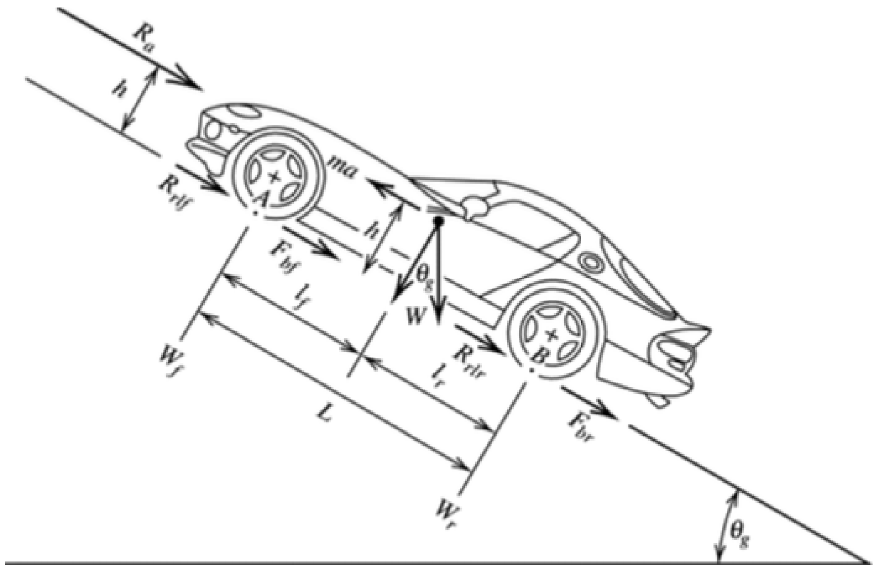


Video

7. PRINCIPLES OF BRAKING

In highway design and traffic analysis, the braking characteristics of road vehicles are arguably the single most important aspect of vehicle performance. The braking behavior of road vehicles is critical in the determination of:

- **stopping sight distance**
- **roadway surface design**
- **accident-avoidance systems**



Braking Forces

$$W_f = \frac{Wl_r cos\theta_g + mah - R_a h \pm Whsin\theta_g}{L} = \frac{1}{L}[Wl_r + h(ma - R_a \pm Wsin\theta_g)]$$

$$W_r = \frac{Wl_f cos\theta_g - mah + R_a h \pm Whsin\theta_g}{L} = \frac{1}{L}[Wl_f - h(ma - R_a \pm Wsin\theta_g)]$$

$$F_b + f_{rl}W = ma - R_a \pm Wsin\theta_g$$

*the contribution of grade resistance (W sin θg) is negative for uphill grades and positive for downhill grades.

7.1. BRAKING FORCES

$$W_f = \frac{1}{L}[Wl_r + h(F_b + f_{rl}W)]$$

$$W_r = \frac{1}{L}[Wl_f - h(F_b + f_{rl}W)]$$

Because the maximum vehicle braking force (F_b max) is equal to the coefficient of road adhesion (μ), multiplied by the vehicle weights normal to the roadway surface,

$$F_{bf_{max}} = \mu W_f = \frac{\mu W}{L}[l_r + h(\mu + f_{rl})]$$

$$F_{br_{max}} = \mu W_r = \frac{\mu W}{L}[l_f - h(\mu + f_{rl})]$$

Table 2.4 Typical Values of Coefficients of Road Adhesion

	Coefficient of road adhesion	
Pavement	Maximum	Slide
Good, dry	1.00*	0.80
Good, wet	0.90	0.60
Poor, dry	0.80	0.55
Poor, wet	0.60	0.30
Packed snow or ice	0.25	0.10

To develop maximum braking forces, the tires should be at the point of an impending slide. If the tires begin to slide (the brakes lock), a significant reduction in road adhesion results. Avoiding this locked condition is the function of antilock braking systems in cars.

7.2. BRAKING FORCE RATIO AND EFFICIENCY

- On a given roadway surface, the maximum attainable vehicle deceleration (using the vehicle's braking system) is equal to μg, where μ is the coefficient of road adhesion and g is the gravitational constant (32.2 ft/s^2).
- To approach this maximum vehicle deceleration, vehicle braking systems must correctly distribute braking forces between the vehicle's front and rear brakes.
- This front-rear proportioning of braking forces (within the vehicle's braking system) will be optimal (achieving a deceleration rate equal to μg) when it is in exactly the same proportion as the ratio of the maximum braking forces on the front and rear axles

$$BFR_{f/r_{max}} = \frac{F_{bf_{max}}}{F_{br_{max}}} = \frac{[l_r - h(\mu + f_{rl})]}{[l_f - h(\mu - f_{rl})]}$$

- the percentage of braking force that the braking system should allocate to the front axle and the back axle for maximum braking is

$$PBF_f = 100 - \frac{100}{1 + BFR_{f/r_{max}}} \qquad and \qquad PBF_r = \frac{100}{1 + BFR_{f/r_{max}}}$$

7.2. BRAKING FORCE RATIO AND EFFICIENCY

EXAMPLE 2.6

A car has a wheelbase of **100 inches** and a center of gravity that is **40 inches** behind the front axle at a height of **24 inches.** If the car is traveling at **80 mi/h** on a road with **poor pavement that is wet**, determine the percentages of braking force that should be allocated to the front and rear brakes (by the vehicle's braking system) to ensure that maximum braking forces are developed.

$$f_{rl} = 0.01\left(1 + \frac{80 \times 5280/3600}{147}\right) = 0.018$$

and $\mu = 0.6$ from Table 2.4 (maximum because we want the tires to be at the point of impending slide). Applying Eq. 2.30 gives

$$\begin{aligned} BFR_{f/r\,max} &= \frac{l_r + h(\mu + f_{rl})}{l_f - h(\mu + f_{rl})} \\ &= \frac{60 + 24(0.6 + 0.018)}{40 - 24(0.6 + 0.018)} \\ &= 2.973 \end{aligned}$$

7.2. BRAKING FORCE RATIO AND EFFICIENCY

EXAMPLE 2.6

A car has a wheelbase of **100 inches** and a center of gravity that is **40 inches** behind the front axle at a height of **24 inches.** If the car is traveling at **80 mi/h** on a road with **poor pavement that is wet**, determine the percentages of braking force that should be allocated to the front and rear brakes (by the vehicle's braking system) to ensure that maximum braking forces are developed.

Using Eq. 2.31, the percentage of the force allocated to the front brakes should be

$$
\begin{aligned}
PBF_f &= 100 - \frac{100}{1 + BFR_{f/r\,max}} \\
&= 100 - \frac{100}{1 + 2.973} \\
&= \underline{\underline{74.83\,\%}}
\end{aligned}
$$

7.2. BRAKING FORCE RATIO AND EFFICIENCY

EXAMPLE 2.6

A car has a wheelbase of **100 inches** and a center of gravity that is **40 inches** behind the front axle at a height of **24 inches.** If the car is traveling at **80 mi/h** on a road with **poor pavement that is wet**, determine the percentages of braking force that should be allocated to the front and rear brakes (by the vehicle's braking system) to ensure that maximum braking forces are developed.

and using Eq. 2.32 (or simply 100 – PBF_f), the percentage of the force allocated to the rear brakes should be

$$PBF_r = \frac{100}{1 + BFR_{f/r\,max}}$$

$$= \frac{100}{1 + 2.973}$$

$$= \underline{\underline{25.17\,\%}}$$

7.2. BRAKING FORCE RATIO AND EFFICIENCY

- The uncertainties in **vehicle weight and road conditions**, vehicle designers often choose a compromise value of brake force proportioning that, on average, provides good braking but is rarely, if ever, optimal.
  - the addition of vehicle cargo and/or passengers will change not only the weight of the vehicle (which affects frl)
  - the distribution of the weight, shifting the height of the center of gravity and its location along the vehicle's longitudinal axis, and this will change the optimal brake force proportioning ($BFR_{f/r\ max}$).
  - changes in road conditions produce different coefficients of adhesion, again changing optimal brake force proportioning.

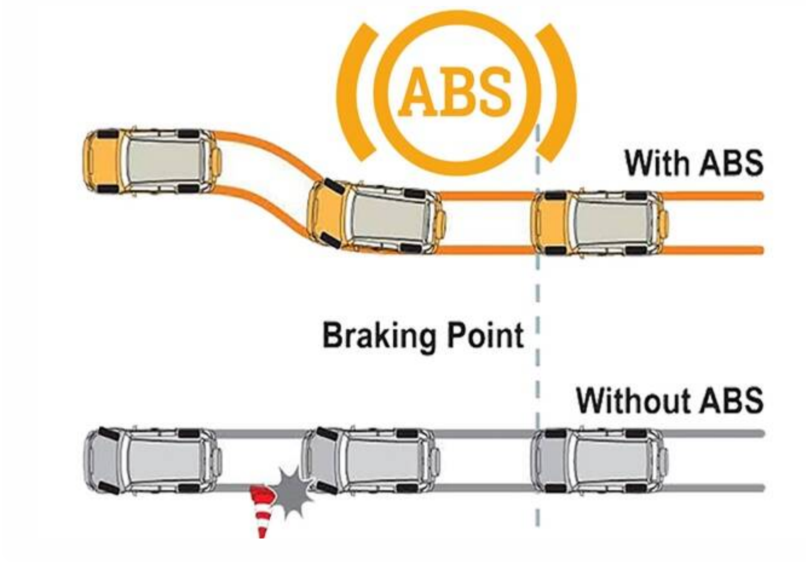
$$\eta_b = \frac{g_{\max}}{\mu} \tag{2.33}$$

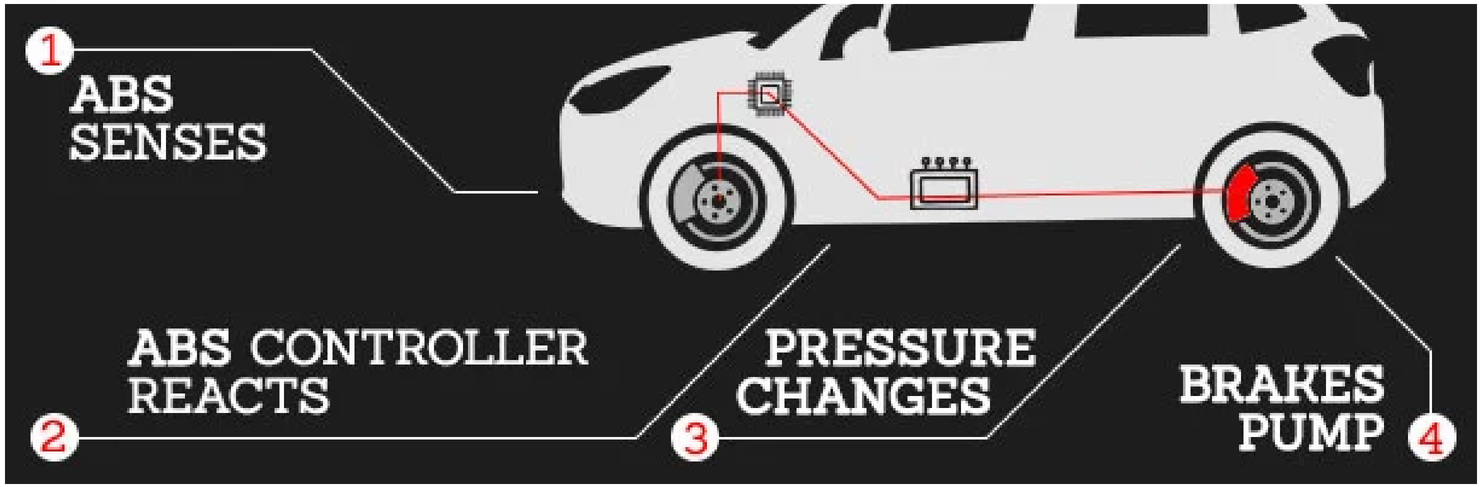
where

η_b = braking efficiency,
g_{max} = maximum deceleration in g units (with the absolute maximum = μ), and
μ = coefficient of road adhesion.

7.3. ANTILOCK BRAKING SYSTEMS

- Many modern cars have braking systems designed to prevent the wheels from locking during braking applications (antilock braking systems). Most antilock braking system technologies detect which wheels have locked and release them momentarily before reapplying the brake on the locking wheel.
- In theory, antilock braking systems serve two purposes:
  - they prevent the coefficient of road adhesion from dropping to slide values
  - they have the potential to raise the braking efficiency to 100%
- It is important to note that studies have indicated that if wheel lockup is to occur, it is preferable to have the front wheels lock first because having the rear wheels lock first can result in uncontrollable vehicle spin.





7.4. THEORETICAL STOPPING DISTANCE

Look at page 32

7.4. THEORETICAL STOPPING DISTANCE

EXAMPLE 2.7

A new experimental **2500-lb** car, with **CD = 0.25** and **Af = 18 ft2**, is traveling at **90 mi/h** down a **10% grade**. The coefficient of road adhesion is **0.7** and the air density is **0.0024 slugs/ft3**. The car has an advanced antilock braking system that gives it a braking efficiency of **100%**. Determine the theoretical minimum stopping distance for the case where aerodynamic resistance is considered and the case where aerodynamic resistance is ignored.

$$f_{rl} = 0.01\left(1 + \frac{\left(\dfrac{90\times 5280/3600+0}{2}\right)}{147}\right) = 0.0145$$

$$K_a = \frac{0.0024}{2}(0.25)(18) = 0.0054$$

Then

$$S = \frac{1.04(2500)}{2(32.2)(0.0054)}\ln\left[1 + \frac{0.0054(90\times 5280/3600)^2}{(1.0)(0.7)(2500)+(0.0145)(2500)-2500\sin(5.71°)}\right]$$

$$= \underline{\underline{444.07\ \text{ft}}}$$

With aerodynamic resistance excluded, Eq. 2.43 is used:

$$S = \frac{1.04(90\times 5280/3600)^2}{2(32.2)(0.7+0.0145-\sin(5.71°))} = \underline{\underline{457.53\ \text{ft}}}$$

$$S = \frac{\gamma_b W}{2gK_a}\ln\left[1+\frac{K_a V_1^2}{\eta_b \mu W + f_{rl}W \pm W\sin\theta_g}\right]$$

$$S = \frac{\gamma_b\left(V_1^2 - V_2^2\right)}{2g\left(\eta_b\mu + f_{rl} \pm \sin\theta_g\right)}$$

7.4. THEORETICAL STOPPING DISTANCE

Homework

A car is traveling at 128.7 km/h and has a braking efficiency of 80%. The brakes are applied to miss an object that is 45.72 m from the point of brake application, and the coefficient of road adhesion is 0.85. Ignoring aerodynamic resistance and assuming the theoretical minimum stopping distance, estimate how fast the car will be going when it strikes the object if (a) the surface is level and (b) the surface is on a 5% upgrade.

*Look for Example 2.8 for guidance

7.4. THEORETICAL STOPPING DISTANCE

EXAMPLE 2.8

A car is traveling at **80 mi/h** and has a braking efficiency of **80%**. The brakes are applied to miss an object that is **150 ft** from the point of brake application, and the coefficient of road adhesion is **0.85**. Ignoring aerodynamic resistance and assuming the theoretical minimum stopping distance, estimate how fast the car will be going when it strikes the object if (a) the surface is level and (b) the surface is on a **5% upgrade.**

In both cases, rolling resistance is approximated as

$$f_{rl} = 0.01\left(1 + \frac{\left(\frac{80\times 5280/3600 + V_2}{2}\right)}{147}\right) = 0.014 + 0.000034V_2$$

Applying Eq. 2.43 for the level grade with $\gamma_b = 1.04$, $\theta_g = 0°$,

$$S = \frac{\gamma_b\left(V_1^2 - V_2^2\right)}{2g\left(\eta_b\mu + f_{rl} \pm \sin\theta_g\right)}$$

$$150 = \frac{1.04\left((80\times 5280/3600)^2 - V_2^2\right)}{2(32.2)\left[0.8(0.85) + (0.014 + 0.000034V_2) \pm 0\right]}$$

$$V_2 = \underline{\underline{85.40 \text{ ft/s}}} \quad \text{or} \quad \underline{\underline{58.23 \text{ mi/h}}}$$

On a 5% grade with $\theta_g = 2.86°$,

$$150 = \frac{1.04\left((80\times 5280/3600)^2 - V_2^2\right)}{2(32.2)\left[0.8(0.85) + (0.014 + 0.000034V_2) + 0.05\right]}$$

$$V_2 = \underline{\underline{82.64 \text{ ft/s}}} \quad \text{or} \quad \underline{\underline{56.35 \text{ mi/h}}}$$

7.5. PRACTICAL STOPPING DISTANCE

- As mentioned earlier, one of the most critical concerns in the design of a highway is the provision of adequate driver sight distance to permit a safe stop.
- The theoretical assessment of vehicle stopping distance presented in the previous section provided the principles of braking for an individual vehicle under specified roadway surface conditions
- highway engineers face a more complex problem because of:
    - a variety of driver skill levels (which can affect whether or not the brakes lock and reduce the coefficient of road adhesion to slide values)
    - vehicle types (with varying aerodynamics, weight distributions, and brake efficiencies)
    - weather conditions (which change the roadway's coefficient of adhesion).
- As a result, the basic physics equation on rectilinear motion, assuming constant deceleration, is chosen as the basis of a practical equation for stopping distance:

$$V_2^2 = V_1^2 + 2ad$$

7.5. PRACTICAL STOPPING DISTANCE

- Rearranging the equation and assuming a is negative for deceleration and the vehicle comes to a complete stop give:

$$V_2^2 = V_1^2 + 2ad \implies 0 = V_1^2 - 2ad \implies d = \frac{V_1^2}{2a}$$

- To account for the effect of grade:

$$d = \frac{V_1^2}{2g[(\frac{a}{g}) \pm G]}$$

- It is important to note the PSD is consistent with TDS (ignores aerodynamic resistance).
- Rewriting TSD with the assumption that the vehicle comes to a stop (V2 = 0), that sin θg = tan θg = G (for small grades), and that γ_b and f_{rl} can be ignored due to their small and essentially offsetting effects,

$$S = \frac{\gamma_b(V_1^2 - V_2^2)}{2g[(\eta_b\mu) + (f_rl) \pm sin\theta]} \implies S = \frac{(V_1^2)}{2g[(\eta_b\mu) \pm G]}$$

- $\eta_b\mu = g_{max}$ However, rather than determining the maximum deceleration rate (in g's) for a specific vehicle braking efficiency and specific coefficient of road adhesion, the AASHTO-recommended maximum deceleration rate (an appropriately conservative value for the overall driver and vehicle population) is used. Thus, a maximum deceleration of **0.35 g's (11.2/32.2)** is used in TPD equation

7.5. PRACTICAL STOPPING DISTANCE

EXAMPLE 2.11

A car [**W = 2200 lb, C_D = 0.25, A_f = 21.5 ft^2**] has an antilock braking system that gives it a **braking efficiency of 100%**. The car's stopping distance is tested on a **level roadway with poor, wet pavement** (with tires at the point of impending skid), and **ρ = 0.00238 slugs/ft3**. How inaccurate will the stopping distance predicted by the practical-stopping-distance equation be compared with the theoretical stopping distance, assuming the car is initially traveling at **60 mi/h**? How inaccurate will the practical-stopping-distance equation be if the same car has a braking efficiency of 85%?

First, to calculate the theoretical minimum stopping distance, Eq. 2.42 is applied with γ_b = 1.04, $\theta_g = 0°$, $\mu = 0.60$ (maximum for poor, wet pavement, from Table 2.4), and

$$f_{rl} = 0.01\left(1 + \frac{\left(\frac{60 \times 5280/3600 + 0}{2}\right)}{147}\right) = 0.013$$

$$K_a = \frac{0.00238}{2}(0.25)(21.5) = 0.0064$$

7.5. PRACTICAL STOPPING DISTANCE

EXAMPLE 2.11

A car [**W = 2200 lb, C_D = 0.25, A_f = 21.5 ft^2**] has an antilock braking system that gives it a **braking efficiency of 100%**. The car's stopping distance is tested on a **level roadway with poor, wet pavement** (with tires at the point of impending skid), and **ρ = 0.00238 slugs/ft3**. How inaccurate will the stopping distance predicted by the practical-stopping-distance equation be compared with the theoretical stopping distance, assuming the car is initially traveling at **60 mi/h**? How inaccurate will the practical-stopping-distance equation be if the same car has a braking efficiency of 85%?

$$S = \frac{1.04(2200)}{2(32.2)(0.0064)} \ln\left[1 + \frac{0.0064(60\times5280/3600)^2}{(1.0)(0.60)(2200)+(0.013)(2200)\ \pm 0}\right] = \underline{\underline{200.35\text{ ft}}}$$

For the same conditions but with a vehicle braking efficiency of 85%, Eq. 2.42 gives

$$S = \frac{1.04(2200)}{2(32.2)(0.0064)} \ln\left[1 + \frac{0.0064(60\times5280/3600)^2}{(0.85)(0.60)(2200)+(0.013)(2200)\ \pm 0}\right]$$
$$= \underline{\underline{234.11\text{ ft}}}$$

Now applying Eq. 2.46 (since $G = 0$) for the practical stopping distance, we find

$$d = \frac{(60\times5280/3600)^2}{2(11.2)} = \underline{\underline{345.71\text{ ft}}}$$

7.5. PRACTICAL STOPPING DISTANCE

EXAMPLE 2.11

A car [**W = 2200 lb, C_D = 0.25, A_f = 21.5 ft^2**] has an antilock braking system that gives it a **braking efficiency of 100%**. The car's stopping distance is tested on a **level roadway with poor, wet pavement** (with tires at the point of impending skid), and **ρ = 0.00238 slugs/ft3**. How inaccurate will the stopping distance predicted by the practical-stopping-distance equation be compared with the theoretical stopping distance, assuming the car is initially traveling at **60 mi/h**? How inaccurate will the practical-stopping-distance equation be if the same car has a braking efficiency of 85%?

Observation

- In the first case, the error is **145.36 ft**. In the case of 85% braking efficiency, the error is **111.60 ft.**
- Rearranging PSD equation to solve for **a**, we find that stopping distances of 200.35 ft and 234.11 ft correspond to deceleration rates of **19.33 ft/s^2 and 16.54 ft/s^2**, respectively.
- Studies [Fambro et al. 1997] have shown that most drivers decelerate at rates of 18.4 ft/s2 or greater in emergency stopping situations.
- Comparing these theoretical values to the AASHTO-recommended deceleration rate of **11.2 ft/s^2**, it is readily apparent that a considerable level of conservatism is built into the deceleration rate for practical stopping distance.

7.6. DISTANCE TRAVELED DURING DRIVER PERCEPTION/REACTION

- It is also necessary to consider the distance traveled during the time the driver is perceiving and reacting to the need to stop.
- The perception/reaction time of a driver is a function of a number of factors:
  - the driver's age
  - physical condition
  - and emotional state
  - the complexity of the situation and the strength of the stimuli requiring a stopping action
- For highway design, a conservative perception/reaction time has been determined to be **2.5 seconds [AASHTO 2011]**. For comparison, average drivers have perception/reaction times of approximately **1.0 to 1.5 seconds.**
- The distance traveled during perception/reaction (d_r) is given by

$$d_r = V_1 + t_r$$

- The total required stopping distance is a combination of the braking distance, either theoretical or practical , and the distance traveled during perception/reaction:

$$d_s = d + d_r$$

7.6. DISTANCE TRAVELED DURING DRIVER PERCEPTION/REACTION

EXAMPLE 2.12

Two drivers each have a reaction time of **2.5 seconds**. One is obeying a **55-mi/h** speed limit and the other is traveling illegally at **70 mi/h**. How much distance will each of the drivers cover while perceiving/reacting to the need to stop, and what will the total stopping distance be for each driver (using practical stopping distance and assuming **G = −2.5%**)?

The distances traveled by each driver during perception/reaction will be calculated first, using Eq. 2.49. For the driver traveling at 55 mi/h,

$$d_r = V_1 \times t_r = (55 \times 5280/3600)(2.5) = \underline{\underline{201.67 \text{ ft}}}$$

For the driver traveling at 70 mi/h,

$$d_r = V_1 \times t_r = (70 \times 5280/3600)(2.5) = \underline{\underline{256.67 \text{ ft}}}$$

Therefore, driving at 70 mi/h increases the distance traveled during perception/reaction by 55.0 ft.

7.6. DISTANCE TRAVELED DURING DRIVER PERCEPTION/REACTION

EXAMPLE 2.12

Two drivers each have a reaction time of **2.5 seconds**. One is obeying a **55-mi/h** speed limit and the other is traveling illegally at **70 mi/h**. How much distance will each of the drivers cover while perceiving/reacting to the need to stop, and what will the total stopping distance be for each driver (using practical stopping distance and assuming **G = –2.5%**)?

Next, the distance traveled during braking will be calculated for each driver, using the equation for practical stopping distance (Eq. 2.47). For the driver traveling at 55 mi/h,

$$d = \frac{(55 \times 5280/3600)^2}{2(32.2)\left(\left(\frac{11.2}{32.2}\right) - 0.025\right)} = \frac{6507.11}{20.79} = 312.99 \text{ ft}$$

For the driver traveling at 70 mi/h,

$$d = \frac{(70 \times 5280/3600)^2}{2(32.2)\left(\left(\frac{11.2}{32.2}\right) - 0.025\right)} = \frac{10540.44}{20.79} = 507.00 \text{ ft}$$

7.6. DISTANCE TRAVELED DURING DRIVER PERCEPTION/REACTION

EXAMPLE 2.12

Two drivers each have a reaction time of **2.5 seconds**. One is obeying a **55-mi/h** speed limit and the other is traveling illegally at **70 mi/h**. How much distance will each of the drivers cover while perceiving/reacting to the need to stop, and what will the total stopping distance be for each driver (using practical stopping distance and assuming **G = –2.5%**)?

The total stopping distance for each driver is now calculated with Eq. 2.50. For the driver traveling at 55 mi/h,

$$d_s = d + d_r = 312.99 + 201.67 = \underline{\underline{514.66 \text{ ft}}}$$

For the driver traveling at 70 mi/h,

$$d_s = d + d_r = 507.00 + 256.67 = \underline{\underline{763.67 \text{ ft}}}$$

Therefore, driving at 70 mi/h increases the total stopping distance by a very substantial 249.01 ft.

QUIZ

A 2500-lb vehicle has a drag coefficient of 0.35 and a frontal area of 20 ft^{2}. **What is the minimum tractive effort required for this vehicle to maintain a 70 mi/h speed** on a 5% upgrade through an air density of 0.002045-slugs/ft3?

a. 217.9 lb
b. 135.1 lb
c. 136.9 lb
d. 172.0 lb